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Use of relativistic hadronic mechanics for the exact representation of nuclear magnetic moments and the prediction of new recycling of nuclear waste

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Summary. We present a new realization of relativistic hadronic mechanics and its underlying iso-Poincaré symmetry specifically constructed for nuclear physics which: 1) permits the representation of nucleons as extended, nonspherical and deformable charge distributions with alterable magnetic moments yet conventional angular momentum and spin; 2) results to be a nonunitary "completion" of relativistic quantum mechanics much along the EPR argument; yet 3) is axiom-preserving, thus preserves conventional quantum laws and the axioms of the special relativity. We show that the proposed new formalism permits the apparently first exact representation of the total magnetic moments of new-body nuclei under conventional physical laws. We then point out that, if experimentally confirmed the alterability of the intrinsic characteristics of nucleons would imply new forms of recycling nuclear waste by the nuclear power plants in their own site, thus avoiding its transportation and storage in a (yet unidentified) dumping area. A number of possible, additional basic advances are also indicated, such as: new understanding of nuclear forces with nowel nonlinear, nonlocal and nonunitary terms due to mutual penetrations of the hyperdense nucleons; consequential new models of nuclear structures; new magnetic confinement of the controlled fusion taking into account the possible alterability of the intrinsic magnetic moments of nucleons at the initiation of the fusion process; new sources of energy based on subnuclear processes; and other possible advances. The paper ends with the proposal of three experiments, all essential for the continuation of scientific studies and all of basic character, relatively moderate cost and

full feasibility in any nuclear physical laboratory.

1 Open character of total nuclear magnetic moments.

One of the fundamental, still unsolved problems of contemporary nuclear physics is the exact representation of the total magnetic moments of nuclei, particularly those of few-body nuclei such as the deuteron, tritium and helium in view of their known limited number of free parameters.

As an example, the experimental value of the deuteron magnetic moment is given by

$$\mu_{\mathrm{D}}^{\mathrm{exp}} = 0.857406 \,, \tag{1.1}$$

while its representation via nonrelativistic quantum mechanics (QM) on D-states yields the familiar epression (see, e.g., [1]).

$$\mu_{\mathrm{QM}}{}^{\mathrm{theor}} = g_N + g_p = 0.880 \,, \tag{1.2}$$

(where g_p and g_n are the gyromagnetic factors of the protons and neutron, respectively) which is about 26 % off *in excess* of the experimental value.

It is known that the use of all possible correction via relativistic quantum mechanics (RQM) on a mixture of S-, D- and P-states can reduce the above deviation down to about 1 %, but RQM *cannot* produce an exact representation of $\mu_{\mathrm{D}}^{\mathrm{exp}}$, as confirmed by the recent studies [2].

It is equally known that the remaining 1 % deviation cannot be eliminated via quark theories because, unlike the corresponding case in the atomic structure, the quark orbits are very small, and their polarization yields corrections to the total magnetic moment of nucleons of the order of 10^{-3} %.

A similar situation exists for representation of the total magnetic moment of the tritium helium and other few-body nuclei.

The most plausible explanation of the above occurrence was formulated by the Founding Father of nuclear physics in the late 1940's immediately after the identification of the numerical value (1.1). For instance, in p. 31 of [1] one can read: *"It is possible that the intrinsic magnetism of nucleon is different when it is in close proximity to another nucleon."*

Recall that nucleons are not point like, but have an extended charge distribution with the radius of about 1 fm (10^{-13} cm). Since perfectly rigid bodies do not exist in the universe, it is plausible to expect that such distribution can be deformed under sufficient external forces. But the deformation of a charged and spinning sphere implies a necessary alteration of its intrinsic magnetic moment, as established by classical electrodynamics.

The above "historical hypothesis" (as referred to hereon) therefore assumes that, when a proton and a neutron are coupled into the deuteron or other nuclear structure, their charge distributions are altered by the nuclear force, resulting in an alteration of their conventional *intrinsic* magnetic moments as measured in vacuum. In turn the assumption of a departure from standard quantum values of the magnetic moments of nucleons readily permits an exact representation of the total magnetic moment of few-body nuclei, as we shall see in Sect. 4.

Since no exact representation of $\mu_D^{\rm exp}$ has been achieved via *conventional* intrinsic magnetic moments of nucleons following about three-quarter of a century of attempts, $\mu_D^{\rm exp}$ should be assumed to constitute *direct experimental evidence on the alterability of the intrinsic magnetic moments of nucleons in the transition from motion in vacuum to motion within nuclear structure.* Note that the representation of $\mu_D^{\rm exp}$ requires a *decrease* of the intrinsic magnetic moments of nucleons, and that such a decrease can only occur for a *prolate* deformation of nucleons referred to their spin axis.

FIGURE 1. **The historical hypothesis on the alterability of the intrinsic magnetic moments of nucleons.** A schematic view of the historical hypothesis on the deformability of the intrinsic magnetic moments of nucleons which constitutes the main topic of study of this paper. As we hope to illustrate, the above hypthesis may stimulate a new scientific renaissance in nuclear physics because it is beyound realistic means of quantitative representation via the Poincaré symmetry and RQM, thus requiring their structural generalizations with far reaching implications of conceptual, theoretical, experimental and applicative character.

Note finally that the above historical hypothesis is *model independent*, i.e., it applies independently from any assumed structure of the nucleons, and

consists of the geometric deformation of their charge distributions whatever the constituents are.

Additional preliminary experimental evidence on the alterability of the intrinsic magnetic moments of nucleons were conducted from 1975 to 1979 by H. Rauch and his associates [3] via interferometric measures of the 4π *spinorial symmetry* of the neutron. The measures are performed via a familiar perfect crystal which splits a thermal neutron beam into two branches which are then coherently recombined. In one (or both) branches experiments [3] placed an electromagnet calibrated at 7,496 G which, for the *conventional* value of the intrinsic magnetic moment of the neutron, would yield an exact multiple of two complete ($4\pi = 720°$) spin flips, as requested by the Fermi-Dirac character of the neutron and as necessary for a coherent recombination.

In order to improve accuracy, the experimenters filled up the electromagnet gap with Mu-metal sheets which reduce stray fields [3]. While crossing the electromagnet gap, the neutron beam is therefore exposed to the field of 7,496 G as well as to the intense electric and magnetic fields in the vicinity of Mu-metal nuclei. The best interferometric measures date back to 1979 with re-elaboration done in 1981 [3e], and are given by

$$\theta = 715.37° \pm 3.8° , \quad \theta_{\min} = 712.07° , \quad \theta_{\max} = 719.67° . \tag{1.3}$$

Such, they *do not* contain 720° in the minimal and maximal values. However, the deviation is *smaller* than the error and, therefore, the above measures are inconclusive.

Similar measures were conducted in 1975 by S.A. Werner and his associates [3f] although also with unsettled results. To our best knowledge, no additional interferometric measures have been done for the 4π spinorial symmetry of the neutron since 1979, thus indicating the need for final tests which are now permitted in view of the technological advances and improved accuracy occurred since the late 1970's.

Despite the above unsettled character, measures (1.3) are significant, as shown in theoretical studies [4]. In this respect let us recall the (p, q)-*deformations* of Lie algebras first introduced by Santilli [4a] back in 1967 as part of his Ph.D. studies with product $(A, B) = p \times A \times B - q \times B \times A$ of Albert's [4a] *Lie-admissible* type, where p, q, and $p \pm q$ are nonzero parameters and $A \times B$ is the usual associative product.

By using the preceding deformations Eder [4c,4d] has shown that the

alteration of the charge distribution of the neutron caused by the intense *electric* and *magnetic* fields in the vicinity of Mu-metal nuclei could indeed yield "spin fluctuations" with about 1 % deviation of the intrinsic magnetic moment which is precisely in order of magnitude needed for the resolution of the historical problem of total nuclear magnetic moments. Note that the *strong interactions* of Mu-metal nuclei have an irrelevant conribution here because their sectional area along the thermal neutron beam is very small.

Also, all *median* angles measured in tests [3] (with the electromagnet gap filled up with Mu-metal sheets) are *smaller* than the expected 720°. This occurrence was studied by Santilli [4e] via the first (p, q)-deformations of the SU(2) spin algebra and called *angle slow-down effect*. This apparent effect is significant inasmuch as it requires a *decrease* of the standard magnetic moment of the neutron for the (polarized) conditions of the experimental set up which is precisely in line with the *decrease* of the same magnetic moments needed for the interpretation of $\mu_{\mathrm{D}}^{\mathrm{exp}}$, as recalled earlier.

The electric and magnetic fields in the vicinity of Mu-metal nuclei are known and result to be of order (in average) of 20,000 G. The biggest unknown is the *deformability of the charge distribution of neutrons* under known external fields, which can only be established from $\mu_{\mathrm{D}}^{\mathrm{exp}}$ for the deuteron conditions are done in Sect.4, or via interferometric measures for more general conditions.

As noted earlier, 4π-interferometric tests can only measure the deformability of neutrons under the intense *electric* and *magnetic* fields of the Mu-metal (or other heavy) nuclei, but not under the *strong nuclear forces* as occuring in the structure of the deuteron.

However, it is known from classical electrodynamics that a small deformation of a spinning and charged sphere can yield a relatively large change of its magnetic moment. Also, the deformability of the charge distributions of nucleons in the deuteron structure may eventually be due to the electric and magnetic fields of the nucleons themselves. Intense electric and magnetic fields of *large, many-body nuclei* could therefore approximate sufficiently well the electric and magnetic fields of the *two-body* deuteron.

The above aspects, combined with the resolution of the still open historical hypothesis as well as with its implications pointed out in Sect. 5, are sufficient to warrant the study of novel methods for the *exact* representation of $\mu_{\mathrm{D}}^{\mathrm{exp}}$, as well as the finalization of interferometric measures on the 4π-spinorial symmetry of the neutron.

2 Expected lack of exact character of quantum mechanics for the nuclear structure

QM is fully established as being *exactly* valid for the so-called *exterior problems*, here referred to as particles moving in vacuum under action-at-a-distance/potential interactions at sufficiently large mutual distances to allow an effective point-like approximation of their wavepackets and/or charge distributions, as occurring in the atomic structure and electroweak interactions at large. In fact, QM provided an *exact* representation of *all* experimental data available for the systems considered.

FIGURE 2. **Experimental insufficiencies of quantum mechanics in nuclear physics**. QM is *exactly* valid in the atomic structure because it provided an exact representation and understanding of *all* its experimental data. On the same grounds, QM cannot be *exactly* valid for the nuclear structure, because it has been unable to provide an *exact* representation of various experimental data. For instance, total nuclear magnetic moments do not follow QM predictions, but are within minimal and maximal values reproduced in this figure from [1] which motivated the historical hypothesis of Fig. 1. Additional insufficiencies exist for: nuclear forces; nuclear structures; total angular momenta; and other aspects. Needless to say, QM provides an excellent approximation of nuclear data. We are therefore referring to deviations which are generally small, yet they have rather important implications, as indicated in Sects 4 and 5.

By comparison, QM *is not* expected to be exactly valid for the so-called *interior problems*, here referred to particles whose wavepackets or charge distributions cannot be effectively approximated as being point-like because moving at small mutual distances (of the order of 1 fm), as occurring in the structure of nuclei (as well as of hadrons and stars not considered in this paper).

The understanding is that the *approximate* validity of QM in nuclear physics is out of scientific debate. We are therefore referring to expected *small deviations* from QM treatments.

The reasons for the above occurrence are numerous. First, unlike the corresponding atomic case, QM has been *unable* to provide an *exact* representation of *all* nuclear experimental data. The lack of exact representation of total nuclear magnetic moments considered in Sect. 1. is only *one* of several isufficiencies. As an example, Ref. [1] indicates the existence of additional lack of final understanding of: nuclear structures, total nuclear angular momenta, and other aspects.

The above experimental insufficiencies can be established in a rigorous theoretical way via primitive symmetry principles. Computer visualizations of the fundamental summetry of QM, the Galilean symmetry G(3.1) or the Poincaré symmetry P(3.1), establish their *exact* validity for *Keplerian systems*, that is, for systems of particles without collisions admitting their heaviest element in the center (Keplerian center). This confirms the exact character of QM for the atomic structure.

By comparison, *nuclei do not possess nuclei* and, consequently, the Galilei and Poincaré symmetries *cannot* be exact for the nuclear structure. In fact, the lack of Keplerian center requires a necessary breaking of the above symmetries. In turn, any expectation of achieving via QM an *exact* representation of *all* nuclear data under these conditions has no theoretical ground.

Not surprisingly, the latter aspects are deeply linked to the preceding ones. In fact, according to the Galilean and Poincaré symmetries in their conventional realization (see Sect. 3 for a more general realization) *the intrinsic magnetic moment of nucleons is perennial and immutable.* Any quantitative representation of the historical hypothesis of their deformability therefore requires a *necessary* deviation from the above symmetries, thus confirming the mutual compatibility of the two aspects.

Both preceding aspects can be rigorously established on dynamical grounds. QM was established for the characterization of action-at-a-distance interactions solely derivable from a potential and this confirms again its exact validity for the atomic structure, this time on dynamical grounds.

By comparison, nucleons in a nuclear structure are in an average state of mutual penetration of about 10^{-3} parts of their charge distribution [4f]. But hadrons are some of the densest objects measured in a laboratory until now. This indicates the presence in the nuclear force of interactions which are: 1) of *contact*, i.e., of zero-range type; 2) *nonlinear* in the wavefunctions and possibly their derivatives; 3) *nonlocal* of a type requring an integral over the volume of overlapping; 4) *nonpotential* in the sense of violating the condi-

tions to be derivable from a potential or a Hamiltonian; and 5) of cosequential *nonunitary* type. By recalling the strictly action-at-a-distance, linear, local-differential, Hamiltonian and unitary character of QM, the preceding characteristics of the nuclear force due to mutual penetration of the hyperdense charge distribution of nucleons are dramatically beyond any hope of quantitative QM treatment.

It should be stressed again that the above isufficiencies *cannot* be resolved via the transition to quark theories on numerous, independent, experimental and theoretical grounds. Besides their inability to achieve the needed exact representation of *all* nuclear data, the current theories on the hadronic structure are also of action-at-a-distance, linear, local-differential, Hamiltonian and unitary character, thus being unable to represent the above expected characteristics of the nuclear force.

FIGURE 3. **Theoretical insufficiences of quantum mechanics in nuclear phusics.** An illustration of the theoretical impossibility for QM to be *exactly* valid for the nuclear structure due to its lack of Keplerian center which requires a necessary breaking of the Galilean and Poincaré symmetries. In turn, the above occurrence is only a consequence of the fundamental theoretical insufficiencies of QM to represent nucleons as *extended, nonspherical and deformable* charge distribution, as well as the inability to represent the component in the nuclear force expected from their mutual penetration which is of *contact, nonlinear, nonlocal, nonhamiltonian and nonunitary* type. An axiom-preserving broadening of QM and its underlying symmetries capable of providing a quantitative representation of the above characteristics is outlined in Sect. 3 and applied in Sect. 4.

Also, quark theories in their conventional formulation are affected by still unresolved basic problems, such as: the lack of a rigorous confinement of the unobservable quarks as prohibited by Heisenberg's uncertainty principle; the inability of quarks to be a representation of the Poincaré symmetry, thus preventing their mathematical parameters called "masses" from being rigorously defined in our space-time (as the eigenvalues) of the second-order Casimir invariant of P(3.1)); the complete lack of gravity for any nucleus assumed to be made up of quarks because of the impossibility of defining

gravity in current quark theories (gravity is solely defined in our spce-time while quarks are solely defined in a mathematical unitary space without interconnections due to the O'Raifeartaigh theorem or known resolution via supersymmetric models).

Thus, any attempts at shifting open problems in our current description of nucleons in our space-time to other, considerably more serious, open problems in their quark constituents, is a *de facto* abandonment of the search for a deeper understanding of the nuclear force and structure.

This leaves no other choice than the search, conducted in Sect. 3, of a broadening-covering of QM capable of providing a quantitative representation of the nuclear aspects under consideration.

The above aspects can be best illustrated via open problem of the total magnetic moments of few-body nuclei. In fact, any quantitative study of the historical hypothesis herein considered requires the introduction of the following *new notions:*

1) The *extended, nonspherical and deformable shape* of the charge distribution of nucleons, expectedly of spheroidal ellipsoidic character, hereon represented with the quantities ${n_1}^2, {n_2}^2, {n_3}^2, n_k \neq 0,\ k = 1, 2, 3$, which are functions of intensity of external fields and any other needed local2 characteristic. For particles with spin along the third axis, the above quantities represent a spheroidal ellipsoids which are *oblate* for ${n_1}^2 = {n_2}^2 > {n_3}^2$ and *prolate* for ${n_1}^2 = {n_2}^2 < {n_3}^2$. The evident condition of preserving the original volume of nucleons then yields the normalization hereon assumed

$$ {n_1}^2 \times {n_2}^2 \times {n_3}^2 = 1 \,, \quad {n_1}^2 = {n_2}^2 > \text{or} < {n_3}^2 \,. \tag{2.1} $$

[It should be noted that in other cases the normalization ${n_1}^2 + {n_2}^2 + {n_3}^2 = 3$ may be preferable].

2) The *density* of the medium in which motion occurs hereon represented with the functions ${n_4}^2$ which, for the vacuum, is assumed to have the normalized value ${n_4}^2 = 1$, and we shall write

$$ {n_4}^2 = 1 \,, \quad < 1 \,, \ \text{or} > 1 \,. \tag{2.2} $$

As we shall see in Sect. 3, n_4 is in reality the local *index of refraction of light*, thus characterizing the local causal speed.

3) The alteration called *mutation* [6b] of the intrinsic magnetic moment μ_N of nucleons $N = n, p$, hereon expressed with the symbol $\hat{\mu}_N = \hat{\mu}_N(\mu_N,$

${n_\mu}^2, \ldots)$, $\mu = 1, 2, 3, 4$, with $\hat{\mu}_N > \mu_N$ for *oblate* deformations and $\hat{\mu}_N < \mu_N$ for *prolate* ones, where the term "mutation" is preferred over "deformation" to indicate the fact that the underlying methods [6] (see the next sections) are structurally different than the known "quantum deformations" [4] of the current literature.

It is evident that the nonspherical and deformable characteristics (2.1) are beyond any representational capability of QM because the latter can only represent *perfectly spherical and perfectly rigid particles*, as necessary in order not to violate the fundamental rotational symmetry O(3). It should be stressed that the same occurrence persists in second-quantization and related form-factors which cannot represent the main characteristics of the historical hypothesis under study here. By comparison, any real treatment of the historical hypothesis requires *ab initio* the representation of *nonspherical and deformable particles*.

The above limitations of QM are well known to be inherent in the very structure of its fundamental carrier spaces, the Euclidean space $E = E(r, \delta, \mathcal{R})$ with coordinates $r = \{r^k\}$, $k = 1, 2, 3$, and metric $\delta = \mathrm{diag}\,(1, 1, 1)$ over the field of real numbers $\mathcal{R} = \mathcal{R}(n, +, \times)$ and the Minkowski space $M = M(x, \eta, \mathcal{R})$ with coordinates $x = \{x^\mu\}$, $\mu = 1, 2, 3, 4$, and metric $\eta = \mathrm{diag}\,(+1, +1, +1, -1)$ over $\mathcal{R}$. In fact, the basic unit of E, $I = \mathrm{diag}\,\{1, 1, 1\}$ (which is the space component of the unit of M) represents a *perfect and rigid sphere*. Moreover, the theory of deformations is well known to be incompatible with the above spaces, their symmetries and, consequently, QM.

Alternatively, it is easy to see that deviations from the exact 720° in the 4π interferometric measures (1.3) imply a deviation from the familiar *spinorial* component of Dirac's wavefunction,

$$\psi' = R(\theta_3) \times \psi = e^{i\gamma_1\gamma_2\theta_3/2} \times \psi\,,$$

where the γ's are the conventional gamma matrices. This is the very reason why the experiment is called the 4π *spinorial symmetry test*.

In fact, mutations of the intrinsic magnetic moment of nucleons imply a departure from its characterization via Dirac's equation from which a departure from law (2.3) follows. At any rate, for an angle of spin flip different than 720°, spinorial law (2.3) *cannot* represent the physical setting.

The above occurrences then leave no other choice than the search for a suitable covering of QM which is more effective for a quantitative represen-

tation of the nuclear aspects under consideration.

In the next section we introduce a new representation of extended, non-spherical and deformable hadrons with main characteristics (2.1) and (2.2) which implies a covering of the spinorial law (2.3) suitable for the exact representation of 4π interferometric measures of type (1.3) and, therefore, of the magnetic moment of few-body nuclei.

Particularly important is the achievement in the next section, apparently for the first time, of *a generalization of the spinorial law (2.3) without altering the spin of nucleons and other QM laws*. The latter advances are needed to dispel a rather general expectation in the earlier studies in the field that a possible confirmation of data (1.3) would imply a departure from the Fermi-Dirac character of the nucleon, with consequential inconsistency with established nuclear laws, e.g., Pauli's exclusion principle. In fact, departures from conventional spin values are present in the "spin fluctuations" of Ref. [4c], the first $SU_{(p,q)}(2)$ quantum group [4e], the proposed test of Pauli's principle under *external strong interactions* [4f] and are inherent in all subsequent q-deformations [4g,4h].

In this paper we present the application of the new formulations of Sect. 3 for the apparently first, exact representation of the total magnetic moments of few body nuclei. We then show that 4π-interferometric measures can indeed provide its independent verification. We finally point out other applications and expected far reaching implications.

3 Relativistic hadronic mechanics

The insufficiencies of QM for the nuclear structure as well as for interior systems in general have been recognized by various research groups. This has stimulated the appearance in recent decades of various studies on possible structural generalizations of QM.

A first attempt was that initiated by Santilli [4b] with the *parameter* (p, q)-deformations of QM with generalized product $(A, B) = p \times A \times B - q \times B \times A$ of Albert's Lie-admissible type, generalized time evolution $i\dot{A} = (A, H)$, and $SU_{p,q}(2)$ quantum structure [4e,4f].

By the time Biedenharn [4g] and Macfarlane [4h] initiated their studies of the simpler class of $(1, q)$-deformations in the mid 1980's (thereafter followed by a very large number of papers), the author had already abandoned this

line of inquiry because of its rather serious problems of *physical* consistency identified, e.g., in Refs. [5].

In fact, (p, q)-time evolutions are evidently *nonunitary*, i.e., they have the structure $U \times U^\dagger \neq I$. As such, they *are not* invariant under their own time evolution which induces the broader *operator* (P, Q)-deformations [6a,6b] with generalized product $(A\hat{,}B) = A \times P \times B - B \times Q \times A$ also of Albert's Lie-admissible type with $P = q \times (U \times U^\dagger)^{-1}$ and $Q = q \times (U \times U^\dagger)^{-1}$, generalized time evolution $i\dot{A} = (A\hat{,}H)$, and correspondingly broader $SU_{P,Q}(2)$ deformations.

The problematic aspects [5] originate from the fact that, after having been achieved via nonunitary transforms, the latter structures are themselves not form-invariant under further nonunitary transforms, thus lacking the axiomatic consistensy of QM.

More generally, all existing deformations of QM with a nonunitary time evolution [4], including q-, k-, quantum-, (p, q)- and (P, Q)-deformations, when formulated on *conventional Hilbert spaces over conventional fields* have the following rather serious problematic aspects of physical nature [5]: 1) the basic unit is not invariant, thus preventing unambiguous applications to experiments; 2) Hermiticity is not conserved in time, thus preventing the existence of unambiguous observables; 3) special functions and transforms are not unique and invariant, thus implying lack of uniqueness and invariance of numerical predictions and physical laws; and other problems.

Most importantly, *all the preceding deformations imply the violation of the special relativity*, e.g., because the deformed Minkowski space and Poincaré symmetry are not isomorphic to the origin ones (see, e.g., [4j]). This creates the problem of identifying new axioms replacing Einstein's axioms, establishing their axiomatic consistency and, after that, proving them experimentally.

In this paper we shall use a third class of covering formulations [6] which apparently resolve the above problematic aspects, thus permitting quantitative studies with invariant basic unit, invariant Hermiticity-observability, unique and invariant special functions, numerical predictions and physical laws, yet possessing a nonunitary structure as evidently necessary for novelty.

Above all, *the covering formulations presented in this section are based on the central requirement of preserving the axioms of the special relativity at the abstract level and merely realize them in a more general way.*

This illustrates the reasons for our insistence in using terms different than "deformations" [4], such as "mutations" [6].

The emerging theory is known under the name of *hadronic mechanics* (HM), today also known (for certain reasons identified below) as *"isotopic completion" of quantum mechanics*, as originally proposed by the author in [6a,6b] and subsequently studied by numerous researchers (see [7] for independent studies and comprehensive bibliographies), and outlined in the recent monographs [6j,6k].

The formulation which is necessary for the study of the historical hypothesis of Fig. 1, and its application to total nuclear magnetic moments is *relativistic hadronic mechanics* (RHM) or *"isotopic completion"* of RQM. Its study in Refs. [6j,6k] is made for the most general possible mutations as apparently needed for extreme, interior, hadronic and astrophysical conditions.

In this section we shall present the apparently first formulation of RHM specifically conceived for nuclear physics under the crucial condition of representing the historical hypothesis of Fig. 1. In so doing, we shall show, also for the first time, that RHM can provide the above representation while preserving all conventional QM laws, such as Heisenberg's uncertainties, Pauli's exclusion principle, etc.

RHM is constructed via maps of RQM called *isotopies* [6a] from the Greek meaning of being "axiom-preserving" and referred to *maps of any given linear, local-differential and unitary theory into its most general possible nonlinear, nonlocal-integral and nonunitary extensions which are nevertheless capable of reconstructing linearity, locality and unitarity in certain generalized spaces called isospaces, and generalized fields called isofields.*

It then follows that isotopic images of fields, spaces, algebras, etc., are isomorphic to the original structures by conception and construction, and they coincide at the abstract, realization free level, all this as preparatory grounds to preserve Einsteinian axioms of the special relativity. Nevertheless, as we shall see shortly, the two theories are physically inequivalent because connected by *nonunitary transforms*.

Recall that the most dominant aspect of the predicted new terms in the nuclear force is that of *not being representable with a Hamiltonian* and, of being *nonunitary* (otherwise we trivially remain within the class of equivalence of RQM). The best way to construct the foundations of RHM is therefore by subjecting the corresponding foundations of RQM to nonunitary transforms.

The fundamental quantities of RQM are: the basic unit of the underlying

Minkowski space, $I = \mathrm{diag}(\{1,1,1\},1)$ in Euclidean space (say, 1 cm) and the unit of time (say, 1 sec) in dimensionless form with $\hbar = 1$; the basic associative product $A \times B$ among generic quantities A, B (which is the same for all products of RQM, those of: numbers, operators, etc., including the modular action $H \times |\psi\rangle$ of operators H on Hilbert states $|\psi\rangle$); and the fundamental relativistic canonical commutation rules $[p_\mu, x^\nu] = p_\mu \times x^\nu - x^\nu \times p_\mu = -i\delta_\mu{}^\nu \times I$.

Under nonunitary transforms, the above quantities become

$$U \times U^\dagger = \hat{I} = \hat{I}^\dagger \neq I \, , \tag{3.1a}$$

$$I \to \hat{I} = U \times I \times U^\dagger \, , \tag{3.1b}$$

$$A \times B \to \hat{A} \hat{\times} \hat{B} = U \times A \times B \times U^\dagger = \hat{A} \times \hat{T} \times \hat{B} \, , \tag{3.1c}$$

$$U \times [p_\mu, x^\nu] \times U^\dagger = [\hat{p}_\mu \hat{,} \hat{x}^\nu] = \hat{p}_\mu \hat{\times} x^\nu - \hat{x}^\nu \hat{\times} \hat{p}_\mu = -i\delta_\mu{}^\nu \times \hat{I} \, , \tag{3.1d}$$

$$\hat{T} = (U \times U^\dagger)^{-1} = \hat{I}^{-1} \, , \quad \hat{K} = U \times K \times U^\dagger \, , \quad K = A, B, p, x \, . \tag{3.1e}$$

The above new images are then assumed as the fundamental quantities of RHM.

A most dominant aspect of the above nonunitary transforms is that they imply the joint mapping, called *lifting* [6a], of the unit $I \to \hat{I}$ while the product is lifted in an amount which is the *inverse* of that of the unit, $A \times B \to \hat{A} \hat{\times} \hat{B} = \hat{A} \times \hat{T} \times \hat{B}$, under which $\hat{I} = \hat{T}^{-1}$ is the correct left and right unit of the new theory,

$$\hat{I} \hat{\times} \hat{A} = \hat{T}^{-1} \times \hat{T} \times \hat{A} \equiv \hat{A} \hat{\times} \hat{I} = \hat{A} \times \hat{T} \times \hat{T}^{-1} \equiv A \, , \quad \forall A \, , \tag{3.2}$$

which case (only) $\hat{I}$ is called the *isounit* and $\hat{T}$ the *isotopic element* [6a,6b].

The emerging new operator envelope ξ is called *isoassociative* because it verifies the associative law with respect to the isoproduct, $\hat{A} \hat{\times} (\hat{B} \times \hat{C}) = (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{C}$. Note that the new unit $\hat{I}$ is Hermitean and will therefore be assumed hereon to be positive-definite. Under these conditions it is evident that *the original envelope ξ and its isotopic image $\hat{\xi}$ are isomorphic by central objective, $\xi \approx \hat{\xi}$, and the map $\xi \to \hat{\xi}$ is an isotopy. Yet they are physically nonequivalent because nonunitarily related.*

The representation of system with RQM is done via the knowledge of *one operator only*, the Hamiltonian H, *under the tacit assumption of the*

simplest possible basic units $I = \text{diag}(\{1,1,1\},1)$. The representation of systems via RHM requires the knowledge of *two* quantities, the conventional Hamiltonian H to represent conventional potential interactions, and a second quantity, the isounit $\hat{I}$, to represent all nonhamiltonian quantities.

We shall therefore assume hereon the realization of the isounit (for $\hbar = 1$),

$$\hat{I} = \text{diag}(\hat{I}_s, \hat{I}_t) = \text{diag}(\{{n_1}^2, {n_2}^2, {n_3}^2\}, {n_4}^2) \times \hat{\Gamma}(x, \dot{x}, \psi, \partial\psi, \ldots) > 0\,, \quad (3.3a)$$

$$\hat{I}_s = \text{diag}\{{n_1}^2, {n_2}^2, {n_3}^2\} \times \hat{\Gamma}_s(x, \dot{x}, \psi, \partial\psi, \ldots)\,, \hat{I}_t = {n_4}^2 \times \hat{\Gamma}_t(x, \dot{x}, \psi, \partial\psi, \ldots)\,, \quad (3.3b)$$

where $\hat{I}_s$ and $\hat{I}_t$ are called the *space and time isounits*, respectively, the ${n_k}^2$'s are quantities (2.1) representing the shape of the hadron considered, ${n_4}^2$ is the quantity (2.2) representing the density of the medium in which motion occurs, and $\hat{\Gamma}$ is a positive-definite 4×4 matrix representing the contact, nonlinear, nonlocal, nonhamiltonian and nonunitary interactions (as identified in Sect. 2). The functional dependence of the isounit remains completely unrestricted in RHM and must be determined from the characteristics of the case at hand exactly as it is the case of the Hamiltonian in RQM.

It is important to know that identifications (3.3) are made following the historical teaching by Hamilton, Lagrange, Jacobi and other Founders of analytic dynamics according to which one quantity alone (we today call the Hamiltonian or the Lagrangian) simply cannot represent the entire physical reality. For this reason they formulated their analytic equations with *external terms.*

Comprehensive classical studies not reported here for brevity (see the review and bibliography in [6j,6k]) have established that the use of isounits (3.3) is analytically equivalent to external terms and, in fact, they have the same number of independent elements. The reformulation of the external terms via the isounit has resulted to be necessary to preserve Einsteinian axioms of the special relativity beginning at the classical level because it permits the preservation of Lie's theory which would be otherwise lost in favor of the broader Lie-admissible theory [6a].

These classical studies have resulted in a new analytic mechanics, called *isohamiltonian mechanics* which is the unique and unambiguous classical image of the operator mechanics outlined in this section (see [6l] for the latest studies and comprehensive bibliography).

A criticism is at times moved according to which RHM is "too broad" because the isounit can have infinitely possible values. This criticism is evidently equivalent to the statement that RQM is "too broad" because it admits infinitely possible Hamiltonians.

In reality, both the Hamiltonian and the isounits are selected via fully identified methods resulting in all applications considered until now in unique and unambiguous expressions. The Hamiltonian is selected via all conventional criteria which are those of the *exterior problems*, such as mass, charge, potential, etc. The isounit is instead selected on the *new* grounds of the *interior problems*, thus requiring the description of extended, nonspherical and deformable shapes, density/index of refraction and contact interactions which are absent in the QM literature of this century.

At any rate, any quantitative representation of the historical hypothesis of Fig. 1 requires the capability to represent all *infinitely possible different shapes of the same nucleon*, thus requiring for consistency infinitely many possible isounits for *each* given Hamiltonian.

Also, the noninitiated reader should know since these introductory lines that, when an isolated interior system is considered from the outside, *internal nonpotential effects must evidently averaged into constants because they are short range*, by therefore resulting in a mere rescaling of the shape and density terms via the constant factor $\hat{\Gamma}_0 = \langle\hat{\Gamma}\rangle$. This occurrence renders preferable the scale invariant description of the characteristic n-quantities, which will be tacitly adopted hereon.

Once the basic isotopic unit, product and commutation rules are known, the next step is the identification of the axiomatically correct structure of RHM. Extensive studies in this respect completed only recently with the appearance of Ref. [6l] have shown that *RHM is as axiomatically consistent as RQM if and only if the nonunitary maps (3.1) are applied to the totality of the formalism of RQM, without any exception known to the author*. In fact, any mixtures of isotopic structure with conventional QM methods leads to a host of inconsistencies which generally remain undetected by nonexperts in the field.

This implies that the formalism of RQM must be reconstructed in such a way to admit $\hat{I}$, rather than 1, as the correct left and right unit. Thus, numbers, metric spaces, geometries, symmetries, Hilbert spaces, etc., have to be reconstructed in terms of the isoproduct $A\hat{\times}B$ with isounit $\hat{I}$. The construction is simple, yet unique and unambiguous, and is done below for the first

time by deriving each new structure from the single nonunitary map (3.1), under the notation according to which all quantities with a "hat" are computed in generalized spaces and those without are computed in conventional spaces.

3.1 Isofields

The first notion of RQM which must be isotopically lifted in order to achieve invariant units, Hermiticity and numerical predictions is that of the fields of ordinary real numbers $\mathcal{R}(n, +, \times)$ and complex numbers $\mathcal{C}(c, +, \times)$ with conventional sum $a+b$, additive unit 0, multiplication $a \times b$ and multiplicative unit I, $a = n$, c, resulting in the *isofields* [6f] $\hat{\mathcal{R}} = \hat{\mathcal{R}}(\hat{n}, +, \hat{\times})$ and $\hat{\mathcal{C}}(\hat{c}, +, \hat{\times})$ of *isoreal numbers* $\hat{n} = U \times n \times U^\dagger = n \times \hat{I}$ and *isocomplex numbers* $\hat{c} = U \times c \times U^\dagger = c \times \hat{I}$, $n \in \mathcal{R}$, $c \in \mathcal{C}$, $\hat{I} \neq \mathcal{R}, \mathcal{C}$ equipped with the conventional sum $\hat{+} \equiv +$ and related additive unit $\hat{0} \equiv 0$, as well as with the isoproduct and related isounit

$$\hat{a}\hat{\times}\hat{b} = U \times a \times b \times U^\dagger = \hat{a} \times \hat{T} \times \hat{b}\,, \quad \hat{I} = \hat{T}^{-1}\,,$$
$$\hat{I}\hat{\times}\hat{a} \equiv \hat{a}\hat{\times}\hat{I} \equiv \hat{a}\,, \;\; \forall a = n, c\,. \tag{3.4}$$

The important property is that $\hat{\mathcal{R}}$ and $\mathcal{C}$ preserve all axioms of a field [6j]. Thus, the liftings $\mathcal{R} \to \hat{\mathcal{R}}$ and $\mathcal{C} \to \hat{\mathcal{C}}$ are isotopies.

For consistency, all operations on numbers must then be isotopically lifted in a simple yet unique and significant way. We have in this way the following *isosquare, isosquare root, isoquotient, isonorm,* etc. (see [6f] for details).

$$\hat{a}^2 = \hat{a}\hat{\times}\hat{a} = a^2 \times \hat{I}\,, \quad \hat{a}^{\frac{\hat{1}}{2}} = a^{\frac{1}{1}} \times \hat{I}^{\frac{1}{2}}\,,$$
$$\hat{a}\hat{/}\hat{b} = (\hat{a}/\hat{b}) \times \hat{I}\,, \quad \hat{|}\hat{a}| = |a| \times \hat{I}\,, \quad a = n, c\,. \tag{3.5}$$

Thus the tradition statement "$2 \times 2 = 4$" remained unchanged since biblical times has meaning for RQM but has no meaning for RHM because one must identify first the selected unit and product for the operation "2×2" to have sense. This illustrates from the outset the insidious inconsistencies in attempting to appraise the new RHM via the use of old mathematics.

In short, RQM is defined for numbers n whose basic unit is the quantity $+1$ dating back to biblical times. RHM is instead defined for new numbers $\hat{n} = n \times \hat{I}$ which admit arbitrary (positive-definite) units $\hat{I}$. As we shall

see shortly, the introduction of the new isonumbers has deep and intriguing implications, including the possibility of defining *new* symmetries for *conventional* line elements and inner products.

3.2 Iso-Hilbert spaces

The second notion of RQM which must be lifted for consistency is that of conventional Hilbert spaces $\mathcal{H}$ with states $|\psi\rangle, |\phi\rangle, \ldots$, inner product $\langle\phi|\psi\rangle \in \mathcal{C}$ and normalization $\langle\psi|\psi\rangle = 1$, resulting in the *iso-Hilbert space* $\hat{\mathcal{H}}$ [6j] with the following *isostates, isoinner product* and *isonormalization*

$$\langle\hat{\phi}\hat{|}\hat{\psi}\rangle = U \times \langle\phi| \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times |\psi\rangle \times U^\dagger =$$
$$= \langle\hat{\phi}| \times \hat{T} \times |\hat{\psi}\rangle \times \hat{I} \in \hat{\mathbf{C}}\,, \tag{3.6a}$$

$$\langle\hat{\psi}| \times \hat{T} \times |\hat{\psi}\rangle = 1\,, \quad |\hat{\psi}\rangle = U \times |\hat{\psi}\rangle\,, \quad \langle\hat{\phi}| = \langle\phi| \times U^\dagger\,. \tag{3.6b}$$

Note that, again for consistency, the isoinner product must be an isocomplex number, i.e., must have the structure $\hat{c} = c \times \hat{I}$. The new composition is still inner (because $\hat{T}\rangle 0$) and, therefore, $\hat{\mathcal{H}}$ is still Hilbert. Then, $\hat{\mathcal{H}} \approx \mathcal{H}$ and the lifting $\mathcal{H} \to \hat{\mathcal{H}}$ is again an isotopy.

The local isomorphism $\mathcal{H} \approx \hat{\mathcal{H}}$ can also be seen from the following *new invariance law of the conventional Hilbert product* here expressed for $\hat{T}$ independent from the integration variable,

$$\langle\phi|\psi\rangle \times I \equiv \langle\phi| \times |\psi\rangle \times \hat{T} \times \hat{T}^{-1} \equiv \langle\phi| \times \hat{T} \times |\psi\rangle \times \hat{I} \equiv \langle\phi\hat{|}\psi\rangle\,. \tag{3.7}$$

Thus, RHM is based on conventional Hilbert spaces, only realized in a way more general than that of current use. 8

The isotopy $\mathcal{H} \to \hat{\mathcal{H}}$ is equally fundamental for the consistency of the theory. To see it, note that, under the lifting $I \to \hat{I} = \hat{T}^{-1}$ and $A \times B \to \hat{A}\hat{\times}\hat{B} = \hat{A}\hat{\times}\hat{T}\hat{\times}\hat{B}$, the action of an operator H on a state must be isotopic, i.e., of the type $\hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H} \times \hat{T} \times |\hat{\psi}\rangle$ because this is the only one admitting the isounit $\hat{I}\hat{\times}|\hat{\psi}\rangle \equiv |\hat{\psi}\rangle$. Then the formulation of the above expression on a *conventional* Hilbert space with inner product $\langle\hat{\phi}| \times |\hat{\psi}\rangle$ implies the general loss of Hermiticity. In fact, we would have the condition $\{\langle\hat{\phi}|\hat{\times}\hat{H}^\dagger\} \times |\hat{\psi}\rangle =$

$\langle\hat{\phi}| \times \{\hat{H}\hat{\times}|\hat{\psi}\rangle\}$, i.e., $\hat{H}^{\hat{\dagger}} = \hat{T}^{-1} \times H^{\dagger} \times \hat{T} \neq \hat{H}^{\dagger}$. On the contrary, the use of the isoHilbert space implies the conditions

$$\{\langle\hat{\phi}|\hat{\times}\hat{H}^{\hat{\dagger}}\}\hat{\times}|\hat{\psi}\rangle = \langle\hat{\phi}|\hat{\times}\{\hat{H}\hat{\times}|\hat{\psi}\rangle\} , \quad \text{i.e.} , \quad \hat{H}^{\hat{\dagger}} = \hat{H}^{\dagger} . \tag{3.8}$$

As a result, *the conditions of Hermiticity and isohermiticity coincide, quantities which are Hermitean-observable for RQM remain so for RHM, the eigenvalues of Hermitean operators of RHM are real*, and other properties (see [6j] from brevity).

The only possible *isoeigenvalues equations* are then given by

$$\begin{aligned} \hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H}(x,p) \times \hat{T}(x,p,\psi,\partial\psi,\ldots) \times |\hat{\psi}\rangle = \hat{E}\hat{\times}|\hat{\psi}\rangle = \\ = (E \times \hat{I}) \times \hat{T} \times |\hat{\psi}\rangle = E \times |\hat{\psi}\rangle . \end{aligned} \tag{3.9}$$

with corresponding *isotopic expectation values*

$$\langle\hat{H}\rangle = \frac{\langle\hat{\psi}|\hat{\times}\hat{H}\hat{\times}|\hat{\psi}\rangle}{\langle\hat{\psi}|\hat{\times}|\hat{\psi}\rangle} = \frac{\langle\hat{\psi}| \times \hat{T} \times \hat{H} \times \hat{T} \times |\hat{\psi}\rangle}{\langle\hat{\psi}| \times \hat{T} \times |\hat{\psi}\rangle} , \tag{3.10}$$

which can be easily seen to coincide with the isoeigenvalues for the same operator. Note from Eq.s (3.9) that the "final numbers" of RHM to be confronted with experiments are conventional.

The fundamental axioms of RHM, are a simple isotopy of the axioms of RQM here omitted for brevity [6k]. We only mention for future needs the axiom.

The above elements illustrate the main property that *RHM coincides with RQM at the abstract realization-free level* for which, from the positive-definiteness of $\hat{I}$, we have $\mathcal{R} \equiv \hat{\mathcal{R}}$, $\mathbf{C} \equiv \hat{\mathbf{C}}$ and $\mathcal{H} \equiv \hat{\mathcal{H}}$. All other aspects of RHM are constructed following the same lines. Thus, *RHM is not a new theory, but merely a new realization of the abstract axioms of RHM.* These properties then establish the axiomatic consistency of RHM to such an extent that any criticism in its axiomatic structure is *de facto* a criticism on the axiomatic structure of RQM.

Despite the above abstract axiomatic identity, one should keep in mind that, as illustrated in Eq.s (3.1), *RHM and RQM are physically inequivalent because the former is a nonunitary image of the latter.* Moreover, isotopies imply the following mapping of eigenvalues

$$H \times |\psi\rangle = E_0 \times |\psi\rangle \rightarrow H \times T \times |\psi\rangle = E \times |\hat{\psi}\rangle \,, \quad E \neq E_0 \,, \tag{3.11}$$

according to which *the same operator H has different eigenvalues in RQM and RHM*, and this illustrates the nontriviality of the isotopies.

3.3 Isolinearity, isolocality, isounitarity

It is important to see that, despite their physical inequivalence, *RHM preserves the conventional linearity, locality and unitarity of RQM*. To begin, RHM is highly nonlinear in the weavefunctions (and their derivatives), as evident from isoeigenvalues expressions (3.9). Yet, the theory is *isolinear*, i.e., it verifies the linearity conditions in isospace, e.g., for all possible $\hat{a} \in \hat{\mathcal{R}}$ or $\hat{\mathcal{C}}$ and $|\hat{\phi}\rangle, |\hat{\psi}\rangle \in \hat{\mathcal{H}}$, we have the identity

$$\hat{A} \hat{\times} (\hat{a} \hat{\times} |\hat{\psi}\rangle + \hat{b} \hat{\times} |\hat{\phi}\rangle) = \hat{a} \hat{\times} \hat{A} \hat{\times} |\hat{\psi}\rangle + \hat{b} \hat{\times} \hat{A} \hat{\times} |\hat{\phi}\rangle \,, \tag{3.12}$$

A similar situation occurs for *locality*. In fact, RHM is *nonlocal-integral* because interactions of that type are admitted in the $\hat{\Gamma}$'s terms of the isounits, Eq.s (3.3). Nevertheless, RHM is *isolocal*, i.e., it verifies the condition of locality in isospace. In particular, RHM is everywhere local-differential except at the isounit. On more technical grounds, RHM is equipped with a new topology called *Tsagas-Sourlas integro-differential topology* [7f].

By recalling that RQM is strictly local-differential, the above new topology has fundamental physical relevance inasmuch as it permits mathematically rigorous quantitative studies of the nonlocal-integral component of the nuclear force needed to represent the overlapping of the hyperdense charge distributions of nucleons in the nuclear structure [5f].

Next, RQM is said to be *unitary* in the sense that the only allowed transformations are of the unitary type, $U \times U^\dagger = U^\dagger \times U = I$. By comparison, RHM is *nonunitary* because its transformation theory is based on the requirement $W \times W^\dagger = \hat{I} \neq I$. Nevertheless, RHM reconstructs unitarity in isospace, a property called *isounitarity*. In fact, the above nonunitary transforms cal be rewritten in the following *identical* isotopic form

$$W = \hat{W} \times \hat{T}^{1/2} \,, \quad W \times W^\dagger = \hat{I} \neq I \,, \tag{3.13a}$$

$$W \times W^\dagger \equiv \hat{W} \hat{\times} \hat{W}^\dagger = \hat{W}^\dagger \hat{\times} \hat{W} = \hat{I} \, , \tag{3.13b}$$

The necessity of the preceding reformulation is soon established by the fact that,even though derived via nonunitarity transforms, the isotopic structures (3.1) and related properties *are not* invariant under additional nonunitary transforms. However, the needed form-invariance is readily achieved under the isounitary reformulation (3.13) for which

$$\hat{I} \to \hat{I} = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^\dagger = \hat{W} \times \hat{T} \times \hat{T}^{-1} \times \hat{T} \times \hat{W}^\dagger \equiv \hat{I} \, , \tag{3.14a}$$

$$\begin{gathered} \hat{A} \hat{\times} \hat{B} \to \hat{W} \hat{\times} \hat{A} \hat{\times} \hat{B} \hat{\times} \hat{W}^\dagger = \hat{A}' \hat{\times} \hat{B}' \, , \\ \hat{K}' = \hat{W} \hat{\times} \hat{K} \hat{\times} \hat{W}^\dagger \, , \quad \hat{K} v = \hat{A} < \hat{B} \, , \end{gathered} \tag{3.14b}$$

with a corresponding invariance of the condition of isohermiticity and all other properties [6k]. This illustrates again that the lack of application of the isotopies to *any* aspect of RQM implies insidious axiomatic inconsistencies.

Note that under isotransforms (3.14) the isounit and isotopic element remain *numerically invariant*. Note also that the transformation theory of RQM is restricted to transforms verifying the condition $U \times U^\dagger = I$ for a fixed I. Similarly, the isotransforms of RHM are restricted to those verifying the condition $\hat{W} \hat{\times} \hat{W}^\dagger = \hat{I}$, this time, for fixed $\hat{I}$ (because its change would imply the description of a *different* system).

3.4 Isotopic physical laws

In this paper we are presenting the simplest possible branch of RHM, that specifically formulated for applications to nuclear physics via a *diagonal, Hermitean and positive* isounit (3.2). It is easy to see that the above branch does indeed preserve all conventional QM laws.

Recall that *generalizations of RQM which are conventionally nonlinear in the wavefunctions, i.e. of the type* $H(x, p, \psi, \ldots) \times \psi = E \times \psi$ [8] *imply the loss of the superposition principle, with consequential inapplicability to a consistent treatment of composite systems such as nuclei*, besides having additional problematic aspects studied in [6k,9].

RHM is also highly nonlinear in view of the eigenvalue structure (3.9), i.e., $H(x, p) \times \hat{T}(\hat{\psi}, \ldots) \times \hat{\psi} = E \times \hat{\psi}$. However, the mathematical notion of

isolinearity has the important physical implication that *RHM preserves the superposition principle in isospace*, as one can verify. This has the important implication that RHM can indeed be consistently applied to composite systems such as few-body nuclei.

Moreover, conventional nonlinear systems can be *identically* reformulated in the isotopic form, $H(t, r, \psi) \times \psi \equiv H_0(t, r) \times \hat{T}(\psi, \ldots)\rangle\psi = E \times \psi$, by therefore recovering axiomatic consistency in isospace.

Next, it is important to see that *RHM preserves Heisenberg's uncertainty principle*. In fact, from isocommutators (3.1d) we have ($\hbar = 1$)

$$\Delta\hat{r}^i \Delta\hat{p}_j \geq \frac{1}{2}\langle[\hat{r}^i \hat{,} \hat{p}_j]\rangle = \frac{1}{2}\delta^i{}_j \ . \tag{3.15}$$

This establishes that the deviations from Heisenberg's uncertainties predicted by quantum deformations (e.g., of the so-called squeezed states [4j]) can be removed via their reformulation in an invariant isotopic form (see [6k] for details).

Along similar lines, it is possible to prove that the notions of isolocality and isounitarity permit the preservation of *causality under nonlocal-integral forces* (see also [6k] for brevity). The preservation of the Fermi-Dirac statistics and related Pauli's exlusion principle will be indicated shortly. The proof of the preservation of other physical laws will be left to the interested reader.

The preservation of conventional laws can be seen from the fact that the fundamental quantity of representing deviations from RQM, the isounit, preserves all axiomatic properties of the conventional unit I, it is the basic invariant of the new theory and its isoexpectation values recover the conventional value I,

$$\hat{I}^{\hat{n}} = \hat{I} \hat{\times} \hat{I} \hat{\times} \ldots \hat{\times} \hat{I} \equiv \hat{I} \ , \quad \hat{I}^{\hat{\frac{1}{2}}} = \hat{I} \ , \quad \hat{I} \hat{/} \hat{I} \equiv \hat{I} \ , \quad \text{etc.} \tag{3.16a}$$

$$\hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^\dagger \equiv \hat{I} \ , \quad i d\hat{I}/dt = \hat{I} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{O} = \hat{H} - \hat{H} \equiv 0 \ , \tag{3.16b}$$

$$\langle \hat{I} \rangle = \frac{\langle \hat{\psi} | \hat{\times} \hat{I} \hat{\times} | \hat{\psi} \rangle}{\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle} = \frac{\langle \hat{\psi} | \times \hat{T} \times \hat{T}^{-1} \times \hat{T} \times | \hat{\psi} \rangle}{\hat{\psi} | \times \hat{T} \times | \hat{\psi} \rangle} = I \tag{3.16c}$$

The above properties establishes the occurrence with far reaching implications according to which *the validity of conventional QM laws for the*

nuclear structure, such as Heisenberg's uncertainty principle, Pauli's exclusion principle, etc., by no means, imply that the conventional formulation of RQM is the only applicable discipline because exactly the same laws are admitted by the structurally more general RHM.

It should be indicated for completeness that in this paper we are studying the simplest possible realization of RHM, that specifically constructed for the nuclear structure under the condition of preserving conventional physical laws. More general realizations exist [6k,6l], e.g., those still of isotopic type with *nondiagonal* isounit $\hat{I}$, or the more general ones of *genotopic type* with nonhermitean basic unit and related transforms

$$I \rightarrow \hat{I} = U \times I \times W^{\dagger} \neq \hat{I}^{\dagger} \ , \quad U \times U \neq I \ , \quad W \times W^{\dagger} \neq I \ , \tag{3.17}$$

which are particularly suited to represent *irreversibility under open-nonconservative conditions*, or those of *hyperstructural* type where $\hat{I}$ is a *set* of nonhermitean elements, which are particularly suited to represent irreversible biological systems [6k], for which conventional QM laws are not necessarily preserved.

The formulation of RHM presented in this section is intended to describe nucleons when members of a nuclear structure with *conventional* spin verifying *conventional* laws and merely having a *nonspherical-deformable* shape. The more general formulations indicated above are intended for more general physical conditions, such as a neutron in the core of a collapsing star considered as external, and will not be considered in this paper.

3.5 Isotopic realization of "hidden variables" and EPR "completion" of RQM

The reader should be aware that *RHM provides an explicit and concrete realization of the theory of "hidden variables"* [10a], which are actually realized via the *operator* $\lambda = \hat{T}(x, \dot{x}, \hat{}, \partial\psi, \ldots)$ and isoeigenvalues

$$\hat{H} \hat{\times}_{\lambda} |\hat{\psi}\rangle = \hat{H} \times \lambda(x, \dot{x}, \psi, \partial\psi, \ldots) \times |\hat{\psi}\rangle = E_{\lambda} \times |\hat{\psi}\rangle \ . \tag{3.18}$$

In fact, the right modular actions "$H \times \psi$" and "$H \hat{\times} \psi$" lose any distinction at the abstract level and, in this sense, they are evidently "hidden" in the conventional realization.

As a result, *RHM constitutes a form of "completion" of RQM, hereon called "isotopic completion", which results to be much along the celebrated argument by Einstein, Podolsky and Rosen* [10b]. In particular, the completion is permitted by the fact that von Neumann's theorem [10c] and Bell's inequalities [10d] are *inapplicable* (and not "violated") for the isotopic completion due to its *nonunitary* structure.

More specifically, von Neumann theorem is inapplicable because the *same* Hamiltonian H has an infinite number of *different* sets of eigenvalues in RHM, one per each possible isotopic element ("hidden operator") $\langle = \hat{T}$. Bell's inequalities are inapplicable, e.g., because RHM requires a *nonunitary image of Pauli's matrices* (see below for their outline). For detailed studies see ref. [6k], App. 4.C. The classical limit under isotopies is also studies in ref. [10e].

A consequence of the above occurrences is that *all applications of RHM outlined below, including the exact representation of nuclear magnetic moments of Sect. 4, are applications of the "isotopic completion" of RQM much along the celebrated $E = P - R$ argument.*

3.6 Iso-Minkowski spaces

The next notion of RQM which must be isotopically lifted for compatibility with basic structures (3.1) is that of the underlying carrier space, the Minkowskian space $M(x, \eta, \mathcal{R})$ with space-time coordinates $x = \{x^\mu\} = \{r, c_0 t\}$, where c_0 is the speed of light in vacuum, and metric $\eta = \text{diag}\,(1, 1, 1, -1)$ and basic unit $I = \text{diag}\,(\{1, 1, 1\}, 1)$ on $\mathcal{R}$. The listing yields the *iso-Minkowski space* $\hat{M} = \hat{M}(\hat{x}, \hat{\eta}, \hat{\mathcal{R}})$, first proposed by Santilli [6e] in 1983, which is characterized by the lifting of: the coordinates x into the *isocoordinates* $\hat{x} = U \times x \times U^\dagger = x \times \hat{I}$; the basic unit of M into the isounit (here assumed to be diagonal from its Hermiticity), $I \to \hat{I}$, and the lifting of the metric η of the *inverse* of that of the unit, $\eta \to \hat{\eta} = \hat{T} \times \eta$. The basic *isointerval* is in then given by or $\hat{x}, \hat{y} \in \hat{M}$

$$(\hat{x} - \hat{y})^{\hat{2}} = [(\hat{x}^\mu - \hat{y}^\nu) \hat{\times} \hat{N}_{\mu\nu}(x, \dot{x}, \hat{\psi}, \partial\psi, \ldots) \hat{\times} (\hat{x}^\nu - \hat{y}^\nu)] \hat{\times} \hat{I} =$$
$$= (x - y)^{\hat{2}} = [(x^\mu - y^\nu) \times \hat{\eta}_{\mu\nu}(x, \dot{x}, \hat{\psi}, \partial\psi, \ldots) \times (x^\nu - y^\nu)] \times \hat{I} =$$
$$= [(x_1 - y_1)^2 \hat{T}_1{}^1 + (x_2 - y_2)^2 \hat{T}_2{}^2 - (x_3 - y_3)^2 \hat{T}_3{}^3 - (x_4 - y_4)^2 \hat{T}_4{}^4] \times \hat{I} \in \hat{\mathcal{R}}. \quad (3.19a)$$

$$\hat{I} = \text{diag}\,(\{\hat{I}^1{}_1, \hat{I}^2{}_2, \hat{I}^3{}_3\}, \hat{I}^4{}_4) = \hat{T}^{-1} > 0\,,$$

$$\hat{T} = \mathrm{diag}\,(\{\hat{T}_1{}^1, \hat{T}_2{}^2, \hat{T}_3{}^3\}, \hat{T}_4{}^4) > 0\,, \tag{3.19b}$$

where $\hat{N}$ is an *isomatrix*, i.e., a matrix whose elements are isoscalars $\hat{N}_{\mu\nu} = \hat{\eta}_{\mu\nu} \times \hat{I} \in \hat{\mathcal{R}}$ (and, therefore, its operations and products are isotopic) while $\hat{\eta}$ is an ordinary matrix, i.e., with elements $\hat{\eta}_{\mu\nu}$ given by ordinary scalars.

Note from the preceding structure that the use of isocoordinates $\hat{x} = x \times \hat{I}$ is redundant in the isointerval. Nevertheless we shall keep using the scripture $\hat{M} = \hat{M}(\hat{x}, \hat{\eta}, \hat{\mathcal{R}})$ rather than $\hat{M}(x, \hat{\eta}, \hat{\mathcal{R}})$ to recall that the coordinates are computed in isospace with respect to a generalized metric. Note also that the isospace $\hat{M}$ and the underlying isofield $\hat{\mathcal{R}}$ share the same generalized unit $\hat{I}$.

Note finally that $\hat{M}$ constitutes the most general possible invariant with signature $(+, +, +, -)$ with a well behaved, yet arbitrary functional dependence on coordinates, wavefunctions, their derivatives of the needed order, as well as any additional quantity of the interior problem.

For this reason, as shown in details by Aringazin [11], the iso-Minkowski space $\hat{M}(\hat{x}, \hat{\eta}, \hat{\mathcal{R}})$ is said to be *directly universal*, i.e., admitting as particular case all possible signature-preserving generalizations of M (universality), directly in the coordinates of the observer (direct universality). In particular, the iso-Minkowskian metric $\hat{\eta}$ admits as particular cases the Riemannian, Finslerian, non-Desarguesian and all other possible metrics in (3+1)-dimension.

Despite the above arbitrariness, it has been proved that the original (abstract) Minkowskian axioms are preserved under the joint liftings $I \to \hat{I} = \hat{T}^{-1}$ and $\eta \to \hat{\eta} = \hat{T} \times \eta$. Thus, $\hat{M} \approx M$ and the lifting $M \to \hat{M}$ is an isotopy.

The latter results can also be seen via the *new invariance law of the conventional Minkowskian interval* (here expressed for a non-null scalar function n)

$$\begin{aligned}
(x-y)^2 = [(x^\mu - y^\mu) \times \eta_{\mu\nu} \times (x^\nu - y^\nu)] \times I \equiv \\
\equiv [(x^\mu - y^\mu) \times (n^{-2} \times \eta_{\mu\nu}) \times (x^\nu - y^\nu)] \times (n^2 \times I) = \\
= [(x^\mu - y^\mu) \times \hat{\eta}_{\mu\nu} \times (x^\nu - y^\nu)] \times \hat{I} = (x-y)^{\hat{2}}\,,
\end{aligned} \tag{3.20}$$

The new invariance identified by RHM is therefore $[L = \text{length}] \times [I = \text{unit}] = \text{Inv}$. As we shall see shortly, this is the mechanism which permits the preservation of spin and other conventinal laws.

Thus, RHM merely expresses "hidden" degrees of freedom of conventional quantum axioms. These degrees of freedom, expressed via the new invariance laws (3.7) and (3.20) have remained undetected through this century because they required the prior discovery of *new numbers*, those with arbitrary units [6f].

3.7 Isodifferential calculus

Despite the use of the isotransformations theory, dynamical equations on $\hat{M}$ *are not* invariant when expressed in terms of the conventional differential calculus. This has requested the construction of the *isodifferential calculus* [6l] which is characterized by a simple, yet unique and effective isotopy of the conventional calculus based on the following *isodifferential, isoderivative* and related primary properties

$$\hat{d}\hat{x}^\mu = \hat{I}^\mu{}_\alpha \times d\hat{x}^\alpha \ , \quad \hat{\partial}/\hat{\partial}\hat{x}^\mu = \hat{T}_\mu{}^\alpha \times \partial/\partial\hat{x}^\alpha \ , \tag{3.21a}$$

$$\hat{\partial}\hat{x}^\mu \hat{/} \hat{\partial}\hat{x}^\nu = \delta^\mu{}_\nu \times \hat{I} \ , \quad \hat{\partial}\hat{x}^\mu \hat{/} \hat{\partial}\hat{x}_\nu = \hat{I}^{\mu\nu} \times \hat{I} \ , \quad \hat{\partial}\hat{x}_\mu \hat{/} \hat{\partial}\hat{x}^\nu = \hat{T}_{\mu\nu} \times \hat{I} \ , \tag{3.21b}$$

where we have implied identities of the type

$$\hat{x}\hat{\times}|\hat{\psi}\rangle = x \times |\hat{\psi}\rangle \ , \quad (\hat{\partial}\hat{/}\hat{\partial}\hat{x})\hat{\times}|\hat{\psi}\rangle \equiv \hat{\partial}/\hat{\partial}\hat{x}|\hat{\psi}\rangle \ , \quad \text{etc.} \tag{3.22}$$

The above isocalculus has only recently permitted the achievement of an axiomatically consistent and form-invariant characterization of the *isotopic linear momentum operator* [6l] which had escaped identification for over a decade and which can be written ($\hbar = 1$)

$$\hat{p}_\mu \hat{\times} |\hat{\psi}\rangle = p_\mu \times \hat{T}(x, \dot{x}, \psi, \partial\psi, \ldots) \times |\hat{\psi}\rangle = -i\hat{\partial}_\mu |\hat{\psi}\rangle = -i\hat{T}_\mu{}^\alpha \partial_\alpha |\hat{\psi}\rangle \ , \tag{3.23}$$

which does indeed recover the fundamental isocommutation rules (3.1d).

Isomomentum (3.23) is of evident fundamental importance because it permits the explicit construction of the Hamiltonian, symmetries, applications, etc.

The integral calculus also admits a simple isotopy with basic definitions $\hat{\int} = \int \times \hat{T}$ for which $\hat{\int} \hat{\partial}\hat{x} = \hat{x}$. For additional details, one may consult [6l].

3.8 Isofunctional analysis

It has been proved [6j] that the elaboration of data in RHM via ordinary and special functions and transforms is inconsistent because not invariant under the time evolution of the theory. This has required the isotopic lifting of functional analysis we cannot possibly review here [6j].

We merely mention for future use that in the transition from the two-dimensional *iso-Euclidean space* with basic unit $\hat{I} = \text{diag}\,({n_1}^2, {n_2}^2)$ to the *iso-Gauss plane* for the characterization of isotrigonometric functions, the isounit assumes the value $\hat{I}_\theta = n_1 \times n_2$ and $\hat{I}_\phi = n_3$ while angles assume the isotopic value $\hat{\theta} = \theta/n_1 \times n_2$, $\hat{\phi} = \phi/n_3$. This permits the construction of the *isotrigonometric functions*

$$\text{isocos}\,\hat{\theta} = n_1 \times \cos(\theta/n_1 \times n_2)\,, \quad \text{isosin}\,\hat{\theta} = n_2 \times \sin(\theta/n_1 \times n_2)\,, \tag{3.24}$$

with corresponding *isospherical coordinates*

$$x = r\,\text{isosin}\,\phi\,\text{isocos}\,\theta\,, \quad y = r\,\text{isosin}\,\phi\,\text{isocos}\,\hat{\theta}\,, \quad z = r\,\text{isocos}\,\phi\,. \tag{3.25}$$

The *isohyperbolic functions* and other structures are then constructed accordingly. Particular important for application is the *iso-Dirac delta function* $\hat{\delta}(\hat{x})$ which, in general, has no longer a singularity at $\hat{x}$, thus having intriguing conceptual and technical implications in the possible removal of singularities *ab initio* (see [6j], for brevity).

3.9 Lie-Santilli isotheory

The fundamental algebraic structure of RQM, Lie's theory, is linear, local-differential and canonical-unitary. As such, it is insufficient to characterize the desired nonlinear, nonlocal-integral and noncanonical-nonunitary component of the nuclear force due to mutual penetration of the hyperdense charge distributions of nucleons.

The primary isotopies of the original proposal [6a,6b,6d] to build HM were those of Lie's theory, i.e., the isotopies of universal enveloping associative algebras, Lie algebras, Lie groups, representation theory, etc. which are today called *Lie-Santilli isotheory* [7].

Again, by conception and construction, the Lie-Santilli isotheory *is not* a new theory, but merely a *new realization* of the abstract axioms of Lie's theory. Also, recall that all Lie algebras (over a field of characteristic zero) are known from Cartan's classification. Therefore, the isotopies of Lie's theory cannot possibly produce new algebras, and have been constructed instead to produce *novel realizations* of known Lie algebras.

The main lines of the Lie-Santilli isotheory can be summarized as follows. Let $\xi(L)$ be the universal enveloping associative algebra of an n-dimensional Lie algebra L with generators $X = \{X_k\} = \{X_k{}^\dagger\}$, unit I, associative product $X_i \times X_j$, and infinite-dimensional basis I, X_k, $X_i \times X_j$, $i \leq j$, $X_i \times X_j \times X_k$, $i \leq j \leq k$, ... (Poincaré-Birkhoff-Witt theorem), and related exponentiation $e^{iXw} = I + (i \times X \times w)/1! + (i \times X \times w) \times (i \times X \times w)/2! + \ldots$, $w \in \mathcal{R}$.

The *universal enveloping isoassociative algebra* $\hat{\xi}(L)$, first proposed in [6a,6d], is the isotopic image of $\xi(L)$ with isounit $\hat{I}$, the same generators $\hat{X}_k = X_k$ only computed in isospace, isoassociative product $\hat{X}_i \hat{\times} \hat{X}_j$, infinite dimensional isobasis $\hat{I}$, $\hat{X}_k$, $\hat{X}_i \hat{\times} \hat{X}_j$, $i \leq j$, $\hat{X}_i \hat{\times} \hat{X}_j \hat{\times}_s \hat{X}_k$, $i \leq j \leq k$, ... (isotopic Poincaré-Birkhoff-Witt theorem [6a,6d,7c]), and *isoexponentiation*

$$\hat{e}^{i \times X \times w} \equiv \hat{e}^{\hat{i} \hat{\times} \hat{X} \hat{\times} \hat{w}} = \{e^{iX \times \hat{T} \times w}\} \times \hat{I} \,, \tag{3.26}$$

where $w = \{w_k\} \in \mathcal{R}$, and $\hat{w} = x \times \hat{I}$ are the *isoparameters*. The script $\hat{\xi}(L)$ rather than $\hat{\xi}(\hat{L})$ is used in the literature [6,7] because, when $\hat{I}$ is no longer positive-definite, in general $\hat{L} \not\approx [\hat{\xi}(L)]^-$. This perm, its the study of a rather intriguing *unification of all simple, compact and noncompact Lie algebra of the same dimension into one unique isoalgebra* [6j]. Note the uniqueness of isoexponentiation (3.26) as compared to the *lack* of uniqueness of the exponentiation for q- and other deformations [5].

Let L be the Lie algebra homomorphic to the antisymmetric algebra $[\xi(L)]^-$ of $\xi(L)$ over a field $F(a, +, \times)$ of real, complex or quaternionic numbers a with familiar Lie's second theorem $[X_i, X_j] = X_i \times X_j - X_j \times X_i = C_{ij}{}^k \times X_k$. The *Lie-Santilli isoalgebra* is the isospace $\hat{L}$ with elements $\hat{X}_k = X_k = X_k{}^\dagger$ on $\hat{\mathcal{H}}$ over $\hat{F}$ with the *isocommutation rules* [6a,6b,6d]

$$[\hat{X}_i \hat{,} \hat{X}_j] = \hat{X}_i \hat{\times} \hat{X}_j - \hat{X}_j \hat{\times} \hat{X}_i = \hat{C}_{ij}{}^k \hat{\times} \hat{X}_k \,, \tag{3.27}$$

whose brackets satisfy *Lie's axioms in the isotopic form* $[\hat{A} \hat{,} \hat{B}] = -[\hat{B} \hat{,} \hat{A}]$, $[\hat{A} \hat{,} [\hat{B} \hat{,} \hat{C}]] + [\hat{B} \hat{,} [\hat{C} \hat{,} \hat{A}]] + [\hat{C} \hat{,} [\hat{A} \hat{,} \hat{B}]] = 0$, and the *isodifferential rules* $[\hat{A} \hat{\times} \hat{B} \hat{,} \hat{C}] = \hat{A} \hat{\times} [\hat{B} \hat{,} \hat{C}] + [\hat{A} \hat{,} \hat{C}] \hat{\times} \hat{B}$.

Let G be the (connected) Lie transformation group characterized by the "exponentiation" of L into the elements $U(w) = e^{i\times X\times w}$ with familiar laws $U(w) \times U(w') = U(w + w')$, $U(w) \times U(-w) = U(0) = I$. Then the (connected) *Lie-Santilli isotransformation group* $\hat{G}$ is the "isoexponentiation" of $\hat{L}$ according to Eq.s (3.26) with isotopic laws

$$\hat{x}' = \hat{U}(\hat{w})\hat{\times}\hat{x} = \hat{e}^{\hat{i}\hat{\times}\hat{X}\hat{\times}\hat{w}}\hat{\times}\hat{x} =$$

$$= \{e^{i\times X\times\hat{T}\times w}\} \times \hat{I} \times \hat{T} \times \hat{x} = \{e^{i\times X\hat{\times}\hat{T}\times w}\} \times \hat{x} \ , \tag{3.28a}$$

$$\hat{U}(\hat{w})\hat{\times}\hat{U}(\hat{w}') = \hat{U}(\hat{w} + \hat{w}') \ , \quad \hat{U}(\hat{w})\hat{\times}\hat{U}(-\hat{w}) = \hat{U}(\hat{0}) = \hat{I} \ . \tag{3.28b}$$

The nontriviality of the above isotopic theory over the conventional formulation is then established by *the appearance of the isotopic element $\hat{T}$ with an unrestricted functional dependence in the exponent of the group structure*. This guarantees that the Lie-Santilli isotheory has the most general possible nonlinear, nonlocal-integral and nonhamiltonian-nonunitary structure, although reformulated in an identical isolinear, isolocal and isounitary form.

A main difference between the Lie theory and the covering Lie-Santilli isotheory is that the former admits only one formulation, while the latter admits *two* formulations, one in isospace over isofields, and the other given by its *projection* in the original space.

As a general rule, *the Lie and Lie-Santilli theories coincide when formulated in their respective spaces*, and this applies also for weights and the representation theory. However, the projection of the latter in the space of the former shows deviations called *mutations* which will be illustrated shortly.

We are now equipped to indicate the preservation of the Fermi-Dirac character of nucleons under the simplest possible isotopy are considered that characterized by nonunitary transforms (3.1) with a diagonal isounit $\hat{I}$. The problem belongs to the study of the axiom-preserving isotopies $S\hat{U}(2)$ of $SU(2)$-spin initiated by Santilli [6h] (which are differet than the axiom-violating deformations $SU_{p,q}(2)$ initiated in [4e]). The same isotopies are reformulated below apparently for the first time via general rule (3.1), resulting in a new class of isorepresentations of $S\hat{U}(2)$ of rather simple construction and effective applications.

Recall that the regular (two-dimensional) representation of $SU(2)$ is characterized by the conventional *Pauli matrices* σ_k with familiar commutation rules $[\sigma_i, \sigma_j] = 2\times i\times\epsilon_{ijk}\times\sigma_k$ and eigenvalues $\sigma^2\times|\psi\rangle = \sigma_k\times\sigma_k\times|\psi\rangle = 3\times|\psi\rangle$, $\sigma_3 \times |\psi\rangle = \pm 1 \times |\psi\rangle$ on $\mathcal{H}$ over $\mathcal{C}$.

RHM requires the construction of *nonunitary images of Pauli's matrices*, which are here submitted for the first time (within the context of RHM) via the rules

$$\hat{\sigma}_k = U \times \sigma_k \times U^\dagger , \quad U \times U^\dagger = \hat{I} \neq I , \tag{3.29a}$$

$$U = \begin{pmatrix} i \times m_1 & 0 \\ 0 & i \times m_2 \end{pmatrix} , \quad U^\dagger = \begin{pmatrix} -i \times m_1 & 0 \\ 0 & -i \times m_2 \end{pmatrix} ,$$

$$\hat{I} = \begin{pmatrix} {m_1}^2 & 0 \\ 0 & {m_2}^2 \end{pmatrix} , \quad \hat{T} = \begin{pmatrix} {m_1}^{-2} & 0 \\ 0 & {m_2}^{-2} \end{pmatrix} , \tag{3.29b}$$

where the m's are well behaved nowhere null functions, resulting in the *regular iso-Pauli matrices*

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & {m_1}^2 \\ {m_2}^2 & 0 \end{pmatrix} , \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times {m_1}^2 \\ i \times {m_2}^2 & 0 \end{pmatrix} , \quad \hat{\sigma}_3 = \begin{pmatrix} {m_1}^2 & 0 \\ 0 & {m_2}^2 \end{pmatrix} . \tag{3.30}$$

Another realization is given by *nondiagonal unitary transforms*

$$U = \begin{pmatrix} 0 & m_1 \\ m_2 & 0 \end{pmatrix} , \quad U^\dagger = \begin{pmatrix} 0 & m_2 \\ m_1 & 0 \end{pmatrix} ,$$

$$\hat{I} = \begin{pmatrix} {m_1}^2 & 0 \\ 0 & {m_2}^2 \end{pmatrix} , \quad \hat{T} = \begin{pmatrix} {m_1}^{-2} & 0 \\ 0 & {m_2}^{-2} \end{pmatrix} , \tag{3.31}$$

with corresponding *regular iso-Pauli matrices*

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & m_1 \times m_2 \\ m_1 \times m_2 & 0 \end{pmatrix} , \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times m_1 \times m_2 \\ i \times m_1 \times m_2 & 0 \end{pmatrix} ,$$

$$\hat{\sigma}_3 = \begin{pmatrix} {m_1}^2 & 0 \\ 0 & {m_2}^2 \end{pmatrix} . \tag{3.32}$$

or by more general realizations of transforms (3.29a), e.g. with Hermitean nondiagonal isounits $\hat{I}$.

All iso-Pauli matrices of the above regular class verify the following iso-commutators rules and isoeigenvalue equations on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$

$$[\hat{\sigma}_i \hat{,} \hat{\sigma}_j] = \hat{\sigma}_i \times \hat{T} \times \hat{\sigma}_j - \hat{\sigma}_j \times \hat{T} \times \hat{\sigma}_i = 2 \times i \times \epsilon_{ijk} \times \hat{\sigma}_k \tag{3.33a}$$

$$\hat{\sigma}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle = (\hat{\sigma}_1 \hat{\times} \hat{\sigma}_1 + \hat{\sigma}_2 \hat{\times} \hat{\sigma}_2 + \hat{\sigma}_3 \hat{\times} \hat{\sigma}_3) \hat{\times} |\hat{\psi}\rangle = 3 \times |\hat{\psi}\rangle , \tag{3.33b}$$

$$\hat{\sigma}_3 \hat{\times} |\hat{\psi}\rangle = \pm 1 \times |\hat{\psi}\rangle . \tag{3.33c}$$

and this establishes the preservation of the Fermi-Dirac statistics and Pauli's exlusion principle for the nuclear realization of RHM under consideration in this section.

We should note that realization (3.31) is the same as that constructed via the so-called *Klimyk's rule* [6k], according to which

$$\hat{\sigma}_k = \sigma_k \times \hat{I} , \quad [\hat{\sigma}_i \hat{,} \hat{\sigma}_j] = [\sigma_i, \sigma_j] \times \hat{I} = 2 \times i \times \epsilon_{ijk} \times \sigma_k \times \hat{I} , \tag{3.34a}$$

$$\hat{\sigma}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle = 3 \times |\hat{\psi}\rangle , \quad \hat{\sigma}_3 \hat{\times} |\hat{\psi}\rangle = \pm |\hat{\psi}\rangle , \tag{3.34b}$$

although realization (3.29) introduced in this paper is evidently broader.

It should be indicated for completeness that the preservation of conventional values of spin has been specifically *selected* here, because in general the isotopies do not preserve the original eigenvalues. As an illustration, the isoselfscalar invariance of the Hilbert space, Eq.s (3.7), implies the existence of the following *irregular iso-Pauli matrices* [6h]

$$\hat{\sigma}_k = \Delta \times \sigma_k \times \hat{I} , \quad [\hat{\sigma}_i, \hat{\sigma}_j] = \Delta \epsilon_{ijk} \times \hat{\sigma}_k , \tag{3.35a}$$

$$\hat{\sigma}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle = 3 \times \Delta^2 \times |\hat{\psi}\rangle , \quad \hat{\sigma}_3 \hat{\times} |\hat{\psi}\rangle = \pm \Delta \times |\hat{\psi}\rangle , \tag{3.35b}$$

where Δ is a well behaved but arbitrary non-null scalar function usually assumed to be $\Delta = \det \hat{I}$, with evident departure from conventional spin values.

In essence, the Fermi-Dirac character of the nucleons when members of a nuclear structure is experimentally established and any generalization of

RQM for nuclear physics must recover this fundamental characteristics, as done with isorepresentation (3.29).

However, the preservation of such Fermi-Dirac character is far from being established on both theoretical and experimental grounds for the same nucleons in more general physical conditions, e.g., in the core of a collapsing star. The more general irregular isorepresentations of $S\hat{U}(2)$ with generalized spin values have been constructed to initiate quantitative studies of the latter more general physical conditions.

3.10 Iso-Poincaré symmetry

As it is well known, that the Lorentz symmetry $L(3.1)$, the Poincaré symmetry $P(3.1) = L(3.1) \times T(3.1)$ and its spinorial covering $\mathcal{P}(3.1) = SL(2.\mathcal{C}) \times T(3.1)$ are not exact for isoseparation (3.19). Their isotopic images were constructed for the first time by Santilli and called *iso-Lorentz symmetry* $\hat{L}(3.1)$ [6e], *iso-Poincaré symmetry* $\hat{P}(3.1) = \hat{L}(3.1)\hat{\times}\hat{T}(3.1)$ [6g], and *isospinorial covering* $\hat{\mathcal{P}}(3.1) = S\hat{L}(2.\hat{\mathcal{C}})\hat{\times}\hat{T}(3.1)$ [6i]. It was also proved that the latter isosymmetries provide indeed the universal invariance of isoseparation (3.19). Moreover, it has been proved in the literature that the above isosymmetries represent indeed extended, nonspherical and deformable charge distributions under conventional values of spin, and characterize indeed contact, nonlinear, nonlocal, nonhamiltonian and nonunitary interactions as expected in the nuclear force.

The main characteristics of the space-time isosymmetries can be summarized as follows. The basic isotopic structures are the field of isoreal numbers $\hat{\mathcal{R}}(\hat{n}, +, \hat{\times})$ and the iso-Minkowski space $\hat{M}(\hat{x}, \hat{\eta}, \hat{\mathcal{R}})$ equipped with the Tsagas-Sourlas isotopology [7f]. The iso-Poincaré symmetry $\hat{P}(3.1)$ on $\hat{M}$ over $\hat{\mathcal{R}}$ is then constructed via the rules of the Lie-Santilli isotheory [6a,6j,7c, 7h]. This essentially consists in preserving the *conventional* generators and parameters

$$X = \{X_k\} = \{M_{\mu\nu} \times p_\alpha\}\,, \quad M_{\mu\nu} = x_\mu \times p_\nu - x_\nu \times p_\mu\,, \tag{3.36a}$$

$$w = \{w_k\} = \{(\theta, v), a\} \in R\,, \tag{3.36b}$$

and by submitting to isotopies the *operations* constructed of them.

FIGURE 4. **Iso-Keplerian systems.** As it well known, the generators $X = \{X_k\} = X^\dagger$ of a QM space-time symmetry represent *total conserved physical quantities*, such as total energy, total linear momentum, etc. The preservation under isotopies of the same generators X assures *ab initio* the preservation of the same total conservation laws. Since space-time isosymmetries imply additional interactions of contact/zero range type, we can therefore see from the outset that *space-time isosymmetries characterize a new class of bound states, called isokeplerian systems, for which the isotopies of Lie's theory were proposed in the first place [6a,6b]. The new bound systems are characterized by conventional, conserved, total physical quantities, yet with constituents in mutual physical contact, exactly as desired for the nuclear structure.* Computer visualization of the iso-Poincaré symmetry $\hat{P}(3.1)$ then yields the elimination of the heaviest constituent at the center, the Keplerian nucleus, and its replacement with an arbitrary constituent, exactly as occurring in the nuclear structure. The iso-Poincaré symmetry $\hat{P}(3.1)$ studied in this section and its isospinorial covering $\hat{\mathcal{P}}(3.1)$ studied in the next section are therefore expected to permit basic novel advances in nuclear physics studied in Sect.s 4 and 5.

The isotopies considered in this paper preserve conventional connectivity properties. Therefore, connected component of the iso-Poincaré symmetry is $\hat{P}_0(3.1) = S\hat{O}(3.1)\hat{\times}\hat{T}(3.1)$, where $S\hat{O}(3.1)$ is the *connected iso-Lorentz group* [6e] and $\hat{T}(3.1)$ is the group of *isotranslations* [6g], with isotransforms on $\hat{M}(\hat{X}, \hat{\eta}, \hat{R})$,

$$\hat{x}' = \hat{A}(\hat{w})\hat{\times}\hat{x} = \hat{A}(\hat{w}) \times \hat{T}(x, \dot{x}, \psi, \partial\psi, \ldots) \times \hat{x} = \tilde{A}(w) \times \hat{x} \,,$$

$$\hat{A} = \tilde{A} \times \hat{I} \,, \tag{3.37}$$

where the first form is the mathematically correct one, the last form being used for computation simplicity. Note that the use of conventional linear transforms $\hat{x}' = A(w) \times \hat{x}$ would now violate linearity in isospace, besides not yielding the desired symmetry of isoseparation (3.19).

The (connected component of the) *iso-Poincaré group* can be written in terms of isoexponentiations (3.26) as (or can be defined by) [6g]

$$\hat{P}_0(3.1) : \hat{A}(\hat{w}) = \Pi_k \hat{e}^{iX\times w} = (\Pi_k e^{iX\times\hat{T}\times w}) \times \hat{I} = \tilde{A}(w) \times \hat{I} \ . \tag{3.38}$$

The preservation of the original dimension is ensured by the *isotopic Baker-Campbell-Hausdorff Theorem* [6a]. It is easy to see that structure (3.38) forms a connected *Lie-Santilli transformation isogroup*.

To identify the isoalgebra $\hat{p}_0(3.1)$ of $\hat{P}(3.1)$ we use the isodifferential calculus (Sect. 3.7) and isolinear momentum (3.23) which yield the *isocommutation rules* [6g]

$$[\hat{M}_{\mu\nu} \hat{,} \hat{M}_{\alpha\beta}] = i(\hat{\eta}_{\nu\alpha}\hat{M}_{\mu\beta} - \hat{\eta}_{\mu\alpha}\hat{M}_{\nu\beta} - \hat{\eta}_{\nu\beta}\hat{M}_{\mu\alpha} + \hat{\eta}_{\mu\beta}\hat{M}_{\alpha\nu}) \ , \tag{3.39a}$$

$$[\hat{M}_{\mu\nu} \hat{,} \hat{p}_\alpha] = i(\hat{\eta}_{\mu\alpha}\hat{p}_\nu - \hat{\eta}_{\nu\alpha}\hat{p}_\mu) \ , \quad [\hat{p}_\alpha \hat{,} \hat{p}_\beta] = 0 \ , \tag{3.39b}$$

where $[A \hat{,} B] = A \times \hat{T}(x, \psi, \ldots) \times B - B \times \hat{T}(x, \psi, \ldots) \times A$.

The *iso-Casimir invariants* are then lifted into the forms [loc.cit.]

$$C^{(0)} = \hat{I}(x, \dot{x}, \psi, \partial\psi, \ldots) = \hat{T}^{-1} \ , \tag{3.40a}$$

$$C^{(1)} = \hat{p}^{\hat{2}} = \hat{p}_\mu \hat{\times} \hat{p}^\mu = \hat{\eta}^{\mu\nu} \hat{p}_\mu \hat{\times} \hat{p}_\nu \ , \tag{3.40b}$$

$$C^{(3)} = \hat{W}_\mu \hat{\times} \hat{W}^\mu \ , \quad \hat{W}_\mu = \epsilon_{\mu\alpha\beta\rho} \hat{M}^{\alpha\beta} \hat{\times} \hat{p}^\rho \ . \tag{3.40c}$$

The local isomorphism $\hat{p}_0(3.1) \approx p_0(3.1)$ is ensured by the positive-definiteness of $\hat{T}$. Alternatively, the use of the generators in the form $\hat{M}^\mu{}_\nu = x^\mu \times p_\nu - x^\nu \times p_\mu$ yields the *conventional* structure constants under a *generalized* Lie product, as one can verify via the use of properties (3.21). The above local isomorphism is sufficient, per sé, to guarantee the axiomatic consistency of RHM.

The main components of $\hat{P}(3.1)$ are the following:

3.10.A. Isorotations, which are the space components $S\hat{O}(3)$[6e,6g,6h]. They can be computed from isoexponentiations (3.38) and the space components $\hat{T}_{kk}$ of the isotopic element in diagonal form, $\hat{T} = \mathrm{diag}\,(T_{\mu\mu})$, $T_{\mu\mu} = \hat{T}_\mu{}^\nu$, yielding the *isorotations in the (x,y)-plane*

$$x' = x \times \cos(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3) - \hat{y} \times \hat{T}_{11}^{-\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \sin(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3)\,, \quad (3.41a)$$

$$y' = \hat{x} \times \hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{-\frac{1}{2}} \times \sin(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3) + \hat{y} \cos(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3)\,, \quad (3.41b)$$

(see [6k] for general isorotations in all three Euler angles).

As one can easily verify, isorotations (3.41) leave invariant all infinitely possible ellipsoidical deformations of the sphere

$$r^{\hat{2}} = x\hat{T}_{11}x + y\hat{T}_{22}y + z\hat{T}_{33}z = R\,, \quad (3.42)$$

thus confirming the achievement of a representation of the deformation theory via a covering of Lie's theory, as needed for a quantitative representation of the historical hypothesis of Fig. 1.

FIGURE 5. **Isosphere.** A central objective of RHM (from which the new mechanics derived its name [6b]) is the representation of hadrons as they are expected to be in the physical reality: *extended, nonspherical and deformable charge distribution*. In nuclear physics, the representation of these characteristics must be achieved under the condition of preserving conventional values of spin. The achievement of this dual objective is geometrically established by the notion depicted in this figure, the *isosphere*, which maps all infinitely possible ellipsoidical shapes into the *perfect sphere* $r^{\hat{2}} = (r^t \times \delta \hat{\times} r) \times \hat{I}$ in the *iso-Euclidean spaces* $\hat{E}(\hat{r}, \hat{\delta}, \hat{\mathcal{R}})$, $\hat{r} = \{\hat{r}^k\} = \{r^k\}$, $\hat{\delta} = \hat{T}_s \times \delta$, $\delta = \mathrm{diag}\,(1,1,1)$, $\hat{T}_s = \mathrm{diag}\,(\hat{T}_{11}, \hat{T}_{22}, \hat{T}_{33})$, $\hat{I}_s = \hat{T}_s^{-1}$ [6j]. In turn, the reconstruction of the perfect spheridicity assures the preservation of the exact rotational symmetry, $\hat{O}(3) \approx O(3)$ and $S\hat{U}(2) \approx SU(2)$, and , consequently, of conventional values of the orbital and intrinsic angular momenta. In fact, the lifting of the semiaxes of the perfect sphere into those of spheroidal ellipsoids, $1_k \rightarrow \hat{T}_{kk}$, when the related units are lifted of the *inverse* amounts, $1_k \rightarrow \hat{T}_{kk}^{-1}$, implies the preservation of the perfect sphericity. The novel model of nuclear structure permitted by the iso-Poincaré symmetry (Fig. 4) is therefore based on *nucleons represented as isospheres*, which are perfect sphere when represented in isospace $\hat{E}$, but when projected in

our space E are given by all infinitely possible spheroidal ellipsoids, exactly as desired for the historical hypothesis of Fig. 1.

3.10.B Iso-Lorentz boosts, which can be written explicitly in the (3,4)-plane [6e]

$$x^{1\prime} = x^1 \ , \quad x^{2\prime} = x^2 \ , \tag{3.43a}$$

$$x^{3\prime} = x^3 \times \sin h(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times v) - x^4 \times \hat{T}_{33}^{-\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times \cos h(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times v) =$$

$$= \hat{\gamma} \times (x^3 - \hat{T}_{33}^{-\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times \hat{\beta} \times x^4) \ , \tag{3.43b}$$

$$x^{4\prime} = -x^3 \times \hat{T}_{33}^{\frac{1}{2}} 2 \times c_0{}^{-1} \hat{T}_{44}^{-\frac{1}{2}} \times \sin h(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times v) + x^4 \times \cos h(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times v) =$$

$$= \hat{\gamma} \times (x^4 - \hat{T}_{33}^{-\frac{1}{2}} \times \hat{T}_{44}^{-\frac{1}{2}} \times \hat{\beta} \times x^3) \ , \tag{3.43c}$$

where

$$\hat{\beta} = (v_k \times \hat{T}_{kk} \times v_k / c_0 \times \hat{T}_{44} \times c_0)^{\frac{1}{2}} \ , \tag{3.44a}$$

$$\hat{\gamma} = (1 - \hat{\beta}^{\hat{2}})^{-1/2} \ . \tag{3.44b}$$

Note that the above isotransforms are *nonlinear* (in $x, \dot{x}, \psi, \partial\psi, \ldots$), *nonlocal-integral* (e.g., because the factor $\hat{\Gamma}$ in (3.3) can be of the type $\exp \int dv \hat{\psi}^\dagger \hat{\psi}$ representing precisely the overlapping of the wavepackets of the constituents) *and nonunitary* (because the isoexponentiations (3.38) are indeed nonunitary in $\mathcal{H}$), precisely as desired, yet they are formally similar to the Lorentz transforms, as expected from their isotopic character. This also confirms the local isomorphism $S\hat{O}(3.1) \approx SO(3.1)$ [6e].

3.10.C. Isotranslations, which can be written [6g]

$$x' = (\hat{e}^{ip \times a}) \hat{\times} x = x + a \times A(x, \ldots) \ , \quad p' = (\hat{e}^{ip \times a}) \hat{\times} p = p \ , \tag{3.45a}$$

$$A_\mu = \hat{T}_{\mu\mu}^{\ 1/2} + a^\alpha[\hat{T}_{\mu\mu}^{\ 1/2}\hat{,}p_\alpha]/1! + \ldots \tag{3.45b}$$

3.10.D. Isoinversions, expressible in the forms

$$\hat{\pi}\hat{\times}x = \pi \times x = (-r, x^4)\,, \quad \hat{\tau}\hat{\times}x = \tau \times x = (r, -x^4)\,, \tag{3.46}$$

where $\hat{\pi} = \pi \times \hat{I}$, $\hat{\tau} = \tau \times \hat{I}$, and π, τ are the conventional inversion operators; and the

3.10.E. Isoscalar transforms, which are the new transforms

$$\hat{I} \rightarrow \hat{I}' = n^2 \times \hat{I}\,, \quad \hat{\eta} \rightarrow \hat{\eta}' = n^{-2} \times \hat{\eta}\,, \tag{3.47}$$

leaving invariant the conventional or isotopic separation, Eq.s (3.19).

Thus, *the iso-Poincaré symmetry is 11-dimensional*, i.e., it has the 10 conventional parameters, plus the parameters n^2 of isotransforms (3.47). Needless to say, the latter new invariance can also be defined for the conventional Poincaré symmetry which, as such, also acquires 11 dimensions.

The isospinorial covering $\hat{\mathcal{P}}(3.1)$ will be identified in the next section. The construction of the *iso-Galilean symmetry* $\hat{G}(3.1)$ via the isotopies of conventional techniques on constractions is an instructive exercise for the interested reader (see also [6k] for an explicit realization).

3.11 Isospecial relativity

On rigorous scientific grounds, the validity of the conventional formulation of the special relativity is nowadays restricted to motion of particles or electromagnetic waves *in vacuum* with constant maximal causal speed c_0. This is due to the fact that, on one side, it is known since the past century (see, e.g., the studies by Lorentz [12a] and their review by Pauli [12b]) that electromagnetic waves propagate within physical media with a locally varying speed $c = c_0/n_4(x, \ldots)\langle c_0$, as it is the case in our atmosphere, water, plastic, glasses, oil, etc.

On the other side, photons propagating within certain guides with speeds $c = c_0/n_4(x, \ldots)\rangle c_0$ have been experimentally measured [13a,13b], and large masses have been measured in astrophysics to be expelled at speeds bigger than c_0 [13c,13d,13e].

Moreover, wave solutions of *conventional* relativistic equations with *arbitrary* speeds have been recently detected in [13f]. Thus, nowadays the speed of electromagnetic waves is no longer a "universal constant" but a local quantity smaller or bigger than c_0 which assumes the constant value c_0 only in vacuum.

It is evident that the special relativity in its current formulationis *inapplicable* (and not "violated") for locally varying speeds $c = c_0/n_4(x, \ldots)$. In addition to the evident loss of the Lorentz and Poincaré symmetries, the insistence of the applicability of the special relativity under conditions for which it was not meant for, e.g., in water, leads to inconsistencies, such as: the assumption of the speed of light $c = c_0/n_4$ in water as the maximal causal speed implies the violation of the principle of causality because electrons can travel in water faster than the speed of light (Cerenkov light); the assumption of the speed of light c_0 *in vacuum* as the maximal causal speed *in water* to salvage the principle of causality implies the violation of the relativistic addition of speeds for which the sum of two speeds of light in water does not yield the speed of light; and other inconsistencies [6k].

Moreover, the special relativity is also known not to be applicable for the description of deformations, as needed for the historical hypothesis of Fig. 1, and can characterize only linear, local-differential and Hamiltonian-unitary systems, while a primary objective of these studies is a quantitative treatment of the nonlinear, nonlocal and nonunitary component expected in the nuclear force.

The *isospecial relativity* was proposed by Santilli [6e,6g,6k] for: the form-invariant description of arbitrary speeds $c = c_0/n_4$; the characterization of extended-deformable shapes of particles; and the form-invariant description of nonlinear, nonlocal and nonunitary interactions. The isospecial relativity is characterized by the axioms of the conventional formulation merely realized in isominkowski space $\hat{M}$ over $\hat{\mathcal{R}}$ under the iso-Poincaré invariance $\hat{P}(3.1)$. As such, the special and isospecial relativity coincide at the abstract level by conception and construction, as it is the case for all other aspects of RHM.

FIGURE 6. **a) Light cone in physical space, b) light "cone" in physical media, c) isolight cone in isospace.** The conventional light cone is well defined only in empty space where light has the constant speed c_0 (Fig. a). Within physical media the speed of electromagnetic waves is however a

local variable, thus implying evident deformations of the conventional light cone (Fig. B). The iso-Lorentz symmetry (6e) maps the latter deformed surface into the *isolight cone*, which is the perfect cone in iso-Minkowskian space $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ (Fig. c). In a way similar to the isosphere, we have the deformation of the light cone axes $! - \mu \rightarrow \hat{T}_{\mu\mu}$ while the corresponding units are deformed of the *inverse* amount $1_\mu \rightarrow \hat{T}_{\mu\mu}{}^{-1}$, thus preserving the original characteristics of a perfect cone. Such a preservation is then the geometric foundation of the local isomorphism $S\hat{O}(3.1) \approx SO(3.1)$. The axiom-preserving character of the isotopy of the light cone is so strong that even the characteristic angle of the cone remains the conventional one, i.e., *the maximal causal speed in isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ remains the speed of light c_0 in vacuum* [6g] (it should be noted that the proof of this property requires, for consistency, the use of the isotrigonometric and isohyperbolic functions we cannot review here for brevity [6j]). This establishes the capability of all problems addressed in this paper to be formulated in a way compatible with the special relativity, only realized in isospace $\hat{M}$.

Variable speeds of electromagnetic waves propagating within inhomogeneous and anisotropic physical media are geometrically represented in a direct way via the isoseparation on $\hat{M}$ for $\hat{T} = \mathrm{diag}\,(\{n_1{}^{-2}, n_2{}^{-2}, n_3{}^{-2}\}, n_4{}^{-2})$, $n_\mu \neq 0$,

$$x^{\hat{2}} = [x^\mu \hat{\eta}_{\mu\nu}(x, \dot{x}, \ldots) x^\nu] \times \hat{I} =$$

$$= (xx/n_1{}^2 + yy/n_2{}^2 + zz/n_3{}^2 - tt \times c_0{}^2/n_4{}^2) \times \hat{I} \in \hat{\mathcal{R}}\,. \tag{3.48}$$

Its evident universal invariance is given by the iso-Poincaré symmetry $\hat{P}(3.1)$. Propagation within homogeneous and isotropic media is expressed by the new invariance (3.20),

$$x^{\hat{2}} = [x^\mu \hat{\eta}_{\mu\nu}(x, \dot{x}, \ldots) x^\nu] \times \hat{I} =$$

$$= (xx/n^2 + yy/n^2 + zz/n^2 - tt \times c_0{}^2/n^2) \times (n^2 \times I) \equiv x^{\hat{2}}\,, \tag{3.49}$$

which is the fundamental symmetry underlying the waves of arbitrary speeds of ref. [13f].

Note that, despite the formal identity $x^{\hat{2}} \equiv x^2$, *the use of the iso-Poincaré symmetry is necessary for the invariance under arbitrary speeds* $c = c_0/n$. In turn, this implies the activation of the entire isotopic formalism of this section. The nontriviality of the $\hat{P}(3.1)$-invariance is then reflected in the appearance of the function $n^2(x, \ldots)$ in the *arguments* of the isorotations, isolorentz boosts, isotranslations, etc., Eq.s (3.41)-(3.47).

One of the first implications of the isospecial relativity is that of permitting the *representation of locally varying speeds of light via the conventional, abstract axioms of the special relativity* [6e,6g]. This is achieved via the reconstruction of c_0 as the unique and universal maximal causal speed in isospace, while its projection in our space-time can assume any possible speed. In fact, jointly with the change $c_0 \rightarrow c_0/n_4$ the unit changes by the inverse amount $1 \rightarrow n_4$, thus preserving the original value c_0. In this way c_0 is a "universal constant" only in isospace $\hat{M}$, while its *projection* in conventional space-time acquires the local form c_0/n_4.

The compatibility of the isospecial relativity with deformable shapes (indicated from the title of the first proposal [6e]) is evident from the unrestricted character of the functional dependence of the isounit. The nonlinear, nonlocal and nonunitary characters are equally evident from the structure of the iso-Poincaré symmetry.

Intriguingly, we can say that *the special relativity is universally applicable only in isospace over isofields*, because only in this latter case we have one single unique and universal causal speeds c_0, all possible deformed light cones are reduced to the perfect cone in isospace, and nonlinear, nonlocal and nonunitary interactions are identically rewritten in their isolinear, isolocal and isounitary form.

3.12 Isotopic dynamical equations

The *fundamental isorelativistic equations* are uniquely identified by the iso-Poincaré symmetry via its iso-Casimir invariants (3.40) and related isorepresentation theory which we cannot possibly study here for brevity (see ref. [6k] for initial studies). The basic equation is the *second-order isorelativistic equation* which is given by the isoinvariant (3.40b) implemented with the conventional minimal coupling rule to an external electromagnetic field

with four-potential $\hat{A}_\mu(x)$, and realized in terms of the isodifferential calculus (3.21)

$$\{[\hat{p}_\mu + i \times e \times \hat{A}_\mu] \hat{\times} [\hat{p}^\mu + i \times e \times \hat{A}^\mu] + \hat{m}^{\hat{2}}\} \hat{\times} |\hat{\psi}\rangle =$$

$$= \{\hat{\eta}^{\mu\nu} \times [\hat{p}_\mu + i \times e \times \hat{A}_\mu] \times \hat{T} \times [\hat{p}_\mu + i \times e \times \hat{A}_\nu] + (m \times m) \times \hat{I}] \times \hat{T} \times |\hat{\psi}\rangle =$$

$$= \{\hat{\eta}^{\mu\nu}[-i\hat{\partial}_\mu + i \times e \times A_\mu] \times [-i\hat{\partial}_\nu + i \times e \times A_\nu] + m^2\} \times |\hat{\psi}\rangle =$$

$$= \{\hat{I}^\mu{}_\alpha \hat{\eta}^{\alpha\nu}[-i\hat{T}_\mu{}^\gamma \times \partial_\gamma + i \times e \times A_\mu] \times [-i\hat{T}_\nu{}^\delta \times \partial_\delta + i \times e \times A_\nu] + m^2\} \times |\hat{\psi}\rangle = 0 \,. \tag{3.50}$$

A solution for the case of null external field and isounits averaged to constant diagonal elements ${n_\mu}^2$ is given by the *isoplane wave* ($c_0 = 1$)

$$\psi(x) = e^{i\hat{I}^\mu{}_\nu \times p_\mu \times x^\nu} \,, \tag{3.51}$$

which does reproduce isoinvariant (3.40b) for constant p's and n's.

For completeness, we quote here the nonrelativistic equations [6k,6l]

$$i\hat{\partial}_t \hat{\psi} = i\hat{T}_t \partial_t \hat{\psi} = \hat{H} \hat{\times}_s \hat{\psi} = \hat{H} \times \hat{T}_s \times \hat{\psi} =$$

$$= \hat{E} \hat{\times}_s \hat{\psi} = (E \times \hat{I}_s) \times \hat{T}_s \times \hat{\psi} = E \times \hat{\psi} \,, \tag{3.52a}$$

$$\hat{\psi}(t, r) = \{\hat{e}^{iH \times t}\} \hat{\times}_s \psi(0, r) = \{e^{i\hat{H} \times \hat{T}_s \times t}\} \times \psi(0, r) \,, \tag{3.52b}$$

$$i\hat{d}\hat{A}/\hat{d}t = i\hat{I}_t d\hat{A}/dt = \hat{A} \hat{\times}_s \hat{H} - \hat{H} \hat{\times}_s \hat{A} = \hat{A} \times \hat{T}_s \times \hat{H} - \hat{H} \times \hat{T}_s \times \hat{A} \,, \tag{3.52c}$$

$$\hat{A}(t) = \{\hat{e}^{i \times \hat{H} \times t}\} \hat{\times}_s \hat{A}(0) \hat{\times}_s \{\hat{e}^{it \times \hat{H}}\} = \{e^{i\hat{H} \times \hat{T}_s \times t}\} \times \hat{A}(0) \times \{e^{it \times \hat{T}_s \times \hat{H}}\}, \tag{3.52d}$$

$$\hat{p}\hat{\times}_s\hat{\psi} = \hat{p} \times \hat{T}_s \times \hat{\psi} = -i\hat{\nabla}_k\hat{\psi} = -i\hat{T}_k{}^i\nabla_i\hat{\psi} \,, \tag{3.52d}$$

$$[\hat{p}_i\hat{,}\hat{r}^j] = \hat{p}_i\hat{\times}_s\hat{r}^j - \hat{r}^j\hat{\times}_s\hat{p}_i = -i\delta_i{}^j I \,, \quad [\hat{p}_i\hat{,}\hat{p}_j] = [\hat{r}^i\hat{,}\hat{r}^j] \equiv 0 \,, \tag{3.52e}$$

$$\hat{I}_i = \hat{I}_t(t, r\hat{\psi}, \ldots) = n_4{}^2 \times \hat{\Gamma}_t(t, r, \hat{\psi}, \ldots) = \hat{T}_t{}^{-1}\rangle 0 \,, \tag{3.52f}$$

$$\hat{T}_s = \hat{T}_s(t, r, \hat{\psi}, \ldots) = \mathrm{diag}(n_1{}^2, n_2{}^2, n_3{}^2) \times \hat{\Gamma}_s(t, r, \hat{\psi}, \ldots)\rangle 0 = \hat{I}_s{}^{-1} \,. \tag{3.52g}$$

with isoplane-wave solution

$$\hat{\psi}(t, r) = e^{i\times(p_k \times n_k{}^2 \times r_k - E \times n_4{}^2 \times t)} \,, \tag{3.53}$$

which coincides with (3.51) as in the conventional case.

One should note the compatibility of the relativistic and nonrelativistic equations, the "decoupling" in the latter of the isounit into i8s space and time components, as well as the isounitary structure of the time evolution in finite form. The proof that the above dynamical equations are indeed form-invariant under their respective relativistic and nonrelativistic isounitary symmetries is an instructive exercise for the interested reader.

4 Exact representation of nuclear magnetic moments

Once the new formalism of RHM is known, the *exact* representation of the total magnetic moments of few-body nuclei becomes straightforward. Its simplicity and exact character should then be compared with the truly complex calculations of ref. [2] via the conventional relativistic/Bethe-Salpeter theories and its lack of exact character.

The most effective derivation is that via the *iso-Dirac equation*, i.e., the isotopies of the conventional Dirac equation [14a] originating from the linearization of the second-order isorelativistic equation (3.50). This linearization has been studied by a number of authors (see [6k], Ch. 10, for details

and references), although in its general form it implies a *mutation of sp6in* which, quite intriguingly, was first discovered by Dirac himself [14b,14c], although without his knowledge of the essential isotopic structure of his own generalized equation [6k].

In this paper we have to re-inspect the derivation of the iso-Dirac equation and introduce a new form specifically intended to represent the mutation of the intrinsic magnetic moments of nucleons while preserving their angular momentum and spin, as necessary for a study of the historical hypothesis (Sect. 1). For other generalizations representing the (constant, yet) *anomalous* magnetic moments of nucleons one may inspect ref. [14d] and papers quoted therein.

The linearization of second-order isoinvariant (3.40b) requires a *composite isounit* characterized by the tensorial product of two nonunitary transforms, one acting in the *orbital* component and on in the *spin* part, resulting in the 8-dimensional total isounit,

$$(U^{\rm orb} \otimes U^{\rm spin}) \times (U^{\rm orb} \otimes U^{\rm spin})^\dagger = \hat{I}^{\rm tot} =$$

$$= \hat{I}^{\rm orb} \otimes \hat{I}^{\rm spin} = (\hat{T}^{\rm orb})^{-1} \otimes (\hat{T}^{\rm spin})^{-1}\rangle 0 \,. \tag{4.1a}$$

$$\hat{I}^{\rm orb} = {\rm diag}\,({n_1}^2, {n_2}^2, {n_3}^2, {n_4}^2)\,, \quad n_\mu \neq 0\,, \tag{4.1b}$$

$$\hat{I}^{\rm spin} = {\rm diag}\,({m_1}^2, {m_2}^2, -{m_1}^2, -{m_2}^2)\,, \quad n_\mu \neq 0\,, \tag{4.1b}$$

$$U^{\rm spin} = \begin{pmatrix} 0 & W_{2\times 2} \\ {W^\dagger}_{2\times 2} & 0 \end{pmatrix}\,, \quad U^{\dagger{\rm spin}} = \begin{pmatrix} 0 & {W^\dagger}_{2\times 2} \\ W_{2\times 2} & 0 \end{pmatrix}\,, \tag{4.1c}$$

$$W_{2\times 2} = \begin{pmatrix} 0 & m_1 \\ m_2 & 0 \end{pmatrix}\,, \quad {W^\dagger}_{2\times 2} = \begin{pmatrix} 0 & m_2 \\ m_1 & 0 \end{pmatrix}\,, \tag{4.1d}$$

$$\hat{I}^{\rm spin}{}_{2\times 2} = W_{2\times 2} \times {W^\dagger}_{2\times 2} = \begin{pmatrix} {m_1}^2 & 0 \\ 0 & {m_2}^2 \end{pmatrix}\,,$$

$$\hat{T}^{\rm spin}{}_{2\times 2} = \begin{pmatrix} {m_1}^{-2} & 0 \\ 0 & {m_2}^{-2} \end{pmatrix}\,, \tag{4.1e}$$

$$\hat{I}^{d\,\mathrm{spin}}{}_{2\times 2} = -\hat{I}^{\dagger\mathrm{spin}}{}_{2\times 2} = -W^{\dagger}{}_{2\times 2} \times W_{2\times 2} = \begin{pmatrix} -{m_1}^2 & 0 \\ 0 & -{m_2}^2 \end{pmatrix} \,,$$

$$\hat{T}^{d\,\mathrm{spin}}{}_{2\times 2} = \begin{pmatrix} -{m_1}^{-2} & 0 \\ 0 & -{m_2}^{-2} \end{pmatrix} \,. \qquad (4.1f)$$

The linearization of Eq.s (3.50) can then be written

$$(\hat{\eta}^{\rho\sigma} \times \hat{p}_\rho \hat{\times} \hat{p}_\nu + \hat{m}^2) \hat{\times} \hat{\psi} = \qquad (4.2)$$

$$= (\hat{\eta}^{\mu\nu} \times \hat{\gamma}_\mu \times \hat{T}^{\mathrm{spin}} \times \hat{p}_\nu \times \hat{T}^{\mathrm{orb}} - i \times m) \times \hat{T}^{\mathrm{tot}} (\hat{\eta}^{\alpha\beta} \times \hat{\gamma}_\alpha \times \hat{T}^{\mathrm{spin}} \times \hat{p}_\beta \times \hat{T}^{\mathrm{orb}} + i \times m) \times \psi =$$

$$= [\hat{\eta}^{\mu\nu} \times \hat{\eta}^{\alpha\beta} \times \frac{1}{2} \times (\hat{\gamma}_\mu \times \hat{T}^{\mathrm{spin}} \times \hat{\gamma}_\alpha \times \hat{T}^{\mathrm{spin}} +$$

$$+ \hat{\gamma}_\alpha \times \hat{T}^{\mathrm{spin}} \times \hat{\gamma}_\mu \times \hat{T}^{\mathrm{spin}}) \times \hat{p}_\nu \times \hat{T}^{\mathrm{orb}} \times \hat{p}_\beta \times \hat{T}^{\mathrm{orb}} + m \times m] \times \hat{\psi} = 0 \,,$$

resulting in the following form of the *iso-Dirac equation* (apparently introduced here for the first time)

$$(\hat{\eta}^{\mu\nu} \times \hat{\gamma}_\mu \hat{\times}^{\mathrm{spin}} \hat{p}_\nu - i \times \hat{m}^2) \hat{\times}^{\mathrm{orb}} \hat{\psi} =$$

$$= (\hat{\eta}^{\mu\nu} \times \hat{\gamma}_\mu \times \hat{T}^{\mathrm{spin}} \times \hat{p}_\nu \times \hat{T}^{\mathrm{orb}} - i \times m \times m) \times \hat{\psi} = 0 \,. \qquad (4.3)$$

The *isogamma matrices* defined by

$$\{\hat{\gamma}_\mu \hat{,} \hat{\gamma}_\alpha\} = \hat{\gamma}_\mu \times \hat{T}^{\mathrm{spin}} \times \hat{\gamma}_\alpha \times \hat{T}^{\mathrm{spin}} + \hat{\gamma}_\alpha \times \hat{T}^{\mathrm{spin}} \times \hat{\gamma}_\mu \times \hat{T}^{\mathrm{spin}} = 2\hat{\eta}_{\mu\nu} \,, \quad (4.4)$$

and admit the explicit realization

$$\hat{\gamma}_\mu = ({\hat{T}_{\mu\mu}}^{\mathrm{orb}})^{1/2} \times U^{\mathrm{spin}} \times \gamma_\mu \times U^{\dagger\,\mathrm{spin}} \times \hat{I}^{\mathrm{spin}} \,, \qquad (4.5a)$$

$$\hat{\gamma}_k = ({\hat{T}_{kk}}^{\mathrm{orb}})^{1/2} \times \gamma_k \times \hat{I}^{\mathrm{spin}} = {\hat{T}_{kk}}^{1/2} \times \begin{pmatrix} 0 & \hat{\sigma}_k \\ \hat{\sigma}^d{}_k & 0 \end{pmatrix} \times \hat{I}^{\mathrm{spin}} \,, \qquad (4.5b)$$

$$\hat{\gamma}_4 = (\hat{T}_{44}{}^{\rm orb})^{1/2} \times \gamma_4 \times \hat{I}^{\rm spin} = (\hat{T}_{kk}{}^{\rm orb})^{1/2} \times \begin{pmatrix} I^{\rm spin}{}_{2\times 2} & 0 \\ 0 & \hat{I}^{d\,{\rm spin}}{}_{2\times 2} \end{pmatrix} \times \hat{I}^{\rm spin}, \quad (4.5c)$$

$$\hat{\sigma}_k = W \times \sigma_k \times W^\dagger , \quad \hat{\sigma}^d{}_k = -\hat{\sigma}^\dagger{}_k = -W^\dagger \times \sigma_k \times W . \quad (4.5d)$$

with a simple extension to the minimal coupling rule hereon tacitly implied.

The nontriviality of the isotopy is then established by the fact that the isotopic elements $\hat{T}_{\mu\mu}$ enter into the structure of the isotopic gamma matrices.

As one can see, the conventional Dirac equations is defined on conventional Minkowski space with basic unit $I = \text{diag}\,(\{1,1,1\},1)$, thus characterizes the perfect and rigid sphere $\{1,1,1\}$ mowing in vacuum, $n_4 = 1$. The above isodirac equation represents instead all infinitely possible ellipsoidical deformations of the perfect sphere with semiaxes $n_1{}^2$, $n_2{}^2$, $n_3{}^2$ under the volume preserving condition (2.1), $n_1{}^2 \times n_2{}^2 \times n_3{}^2 = 1$, while propagating within a physical media with index of refraction $n_4 \neq 1$.

First, it is important to verify that, despite the alteration of the shape of the charge distribution, the *values of the angular momenta are conventional.* This is easily established by the fact that the isodirac equation (4.3) characterizes the following isotopic $S\hat{O}(3)$ algebra (where all products are referred to the orbital isotopic product)

$$\hat{O}(3) \ : \quad \hat{M}_k = \frac{1}{2}\epsilon_{kij}\hat{\hat{r}}^i \hat{\times} \hat{p}_j \ , \quad (4.6a)$$

$$[\hat{M}_i \hat{,} \hat{M}_j] = \hat{M}_i \times \hat{T} \times \hat{M}_j - \hat{M}_j \times \hat{M}_i = \epsilon_{ijk} \times \hat{M}_k \ , \quad (4.6b)$$

$$\hat{M}^{\hat{2}} \hat{\times} \hat{\psi} = \hat{M}_k \times \hat{T} \times \hat{M}^k \times \hat{T} \times \hat{\psi} = m(m+1) \times \hat{\psi} \ , \quad (4.6c)$$

$$\hat{M}_3 \times \hat{\psi} = \hat{M}_k \times \hat{T} \times \hat{\psi} = (\pm m) \times \hat{\psi} \ , \quad m = 0,1,2,\dots \ . \quad (4.6d)$$

This assures that Eq.s (4.3) characterize conventional eigenvalues of the angular momentum.

Second it is easy to see that the $S\hat{U}(2)$ spin algebra on the isofield $\hat{\mathcal{C}} = \hat{\mathcal{C}}(\hat{c}, +, \hat{\times})$ as characterized by the above isodirac equation has a *generalized* structure, yet *conventional* eigenvalues as desired. In fact, we have the following expressions in terms of spin isoproducts

$$S\hat{U}(2) \; : \quad \hat{S}_k = \frac{1}{2}\epsilon_{kij}\hat{\gamma}_i \hat{\times} \hat{\gamma}_j \; , \tag{4.7a}$$

$$[\hat{S}_i \hat{,} \hat{S}_j] = \hat{S}_i \times \hat{T} \times \hat{S}_j - \hat{S}_j \times \hat{S}_i = \epsilon_{ijk} \times \hat{S}_k \; , \tag{4.7b}$$

$$\hat{S}^{\hat{2}} \hat{\times} \hat{\psi} = \hat{S}_k \times \hat{T} \times \hat{S}^k \times \hat{T} \times \hat{\psi} = (3/4) \times \hat{\psi} \; , \tag{4.7c}$$

$$\hat{S}_3 \times \hat{\psi} = \hat{S}_k \times \hat{T} \times \hat{\psi} = (\pm 1/2) \times \hat{\psi} \; , \tag{4.7d}$$

which constitute a 4×4 extension of results (3.33). This assures the characterization of conventional spin, with consequential preservation of Pauli's exclusion principle.

The combined generators $M = (M_{\mu\nu})$, $M_{ij} = \epsilon_{ijk}S_k$, $M_{k4} = i\hat{\gamma}_k \hat{\times} \hat{\gamma}_4$ then characterize the *isospinorial covering* $S\hat{L}(2.\hat{\mathcal{C}})$ of the iso-Lorentz algebra $\hat{L}(3.1)$. The study of the isocommutation rules and local isomorphism $S\hat{L}(2.\hat{\mathcal{C}}) \approx SL(2.\mathcal{C})$ is left to the interested reader, jointly with the isotopies of the remaining aspects of Dirac's theory [6k].

Iso-Dirac equation (4.3) provides the desired two generalized expressions which are needed for a fit of the experimental data on nuclear magnetic moments. First, Eq. (4.3) implies the following desired *mutation of the spinorial transformation law*, first identified by Santilli in ref. [15]

$$\hat{\psi}' = \hat{R}(\theta_3) \hat{\times} \hat{\psi} = e^{i\gamma_1\gamma_2\hat{\theta}_3/2}\hat{\psi} \; , \quad \hat{\theta} = \theta/n_1 n_2 \; , \tag{4.8}$$

Then, a simple isotopy of the conventional case yields the desired *mutation of the magnetic moment of nucleons* (see [6k], Ch. 10. for details)

$$\hat{\mu}_N = \hat{\mu}_N(\mu_N, {n_\mu}^2) = \mu_N \times n_4/n_3 \; . \tag{4.9}$$

first proposed in [6b]. Sect. 4.20, p. 803.

In summary, *RHM can represent in first-quantization (and without any need of form factors) the extended, nonspherical and deformable character of nucleons and the alteration of their intrinsic magnetic moment while preserving the conventional orbital and intrinsic angular momenta and other physiacal laws*. These conditions are necessary for consistency, evidently because neutrons are under *external* electromagnetic fields for which the angular momenta are preserved.

It is important to apply Eq.s (4.8) and (4.9), first, to the exact representation of 4π-interferometric measures of type (1.3), and then to the exact representation of total magnetic moments of few body nuclei. These results were first presented by Santilli [15] during the meeting ¡Deuteron 1993¿ at the JINR in Dubna, Russia. However, the calculations were done for values of $n_4 \neq 1$ (interpreted as the density of the neutrons), and under a joint mutation of spin. It is important to review these results for the more appropriate interpretation of n_4 introduced in this paper.

Assume that the 4π-mutation is 1%, yielding $\theta = 713°$, which is of the order of magnitude of the measures (1.3). The isotopies re-construct the exact $SU(2)$ symmetry in isospace, thus requiring $\hat{\theta} = \theta / n_1 \times n_2 = 720°$ [6g,6k]. This yields

$$
{n_1}^2 = {n_2}^2 = 713°/720° = 0.990 \,, \quad {n_3}^2 = 1/0.990 \times 0.990 = 1.020 \,, \tag{4.10}
$$

namely, the deformation is given by the transition from a perfectly spherical charge disribution to one of *prolate* character, exactly as needed for the deuteron (Sect. 1).

Note that the different normalization ${n_1}^2 + {n_2}^2 + {n_3}^2 = 3$ (Sect. 2) yields the values

$$
{n_1}^2 = -{n_2}^2 = 0.990 \,, \quad {n_3}^2 = 3 - 2 \times 0.990 = 1.020 \,, \tag{4.11}
$$

which coincide with values (4.10).

Then, assuming in first approximation that $\hat{\mu}/\mu = n_4/n_3 \approx 713°/720°$, we have the remaining value

$$
n_4 = n_3 \times 713°/714° = 1.000 \,, \tag{4.12}
$$

namely, the isodirac equation is capable of *deriving* the value $n_4 = 1$ occurring for motion in vacuum, exactly as it is the case for the thermal neutron beam of tests [3].

We now study the application of isodirac equation (4.3) for the exact-numerical representation of the total magnetic moments of few-body nuclei. For this we assume to a good approximation that protons and neutron have the same size and shape, thus admitting the same ellipsoidical shape with ${n_1}^2 = {n_2}^2 \langle {n_3}^2$ or $\rangle {n_3}^2$, ${n_1}^2 \times {n_2}^2 \times {n_3}^2 = 1$.

In regard to the value of n_4, the motion of the proton and neutron in the deuteron, strictly speaking, *is not* in vacuum because each particle is moving within the wavepackets of the other, thus resulting in a difference of n_4 from 1. However the deuteron size is considerably bigger than the nucleon charge radius. As a result, we can assume *in first approximation* that $n_4 = 1$, with the understanding that a refinement of the data is expected whenever *experimental* information on the value of n_4 for the deuteron is known.

A simple isotopy of the conventional QM model (see, e.g, [1]) then yields the following *isotopic theory for the total nuclear magnetic moments*

$$\hat{\mu}^{\rm tot} = \sum_k (\hat{g}_k{}^L \times \hat{M}_{k3} + \hat{g}_k{}^s \times \hat{S}_{k3}) \ , \qquad (4.13a)$$

$$\hat{g}_n = g_n n_4/n_3 \approx g_n/n_3 \ , \quad \hat{g}_p = g_p n_4/n_3 \approx g_p/n_3 \ , \qquad (4.13b)$$

$$e\hbar/2m_p c_0 = 1 \ , \quad g_n{}^s = -3.816 \ , \quad g_p{}^s = 5.585 \ , \qquad (4.13c)$$

$$g_n{}^M = 0 \ , \quad g_p{}^M = 1 \ , \qquad (4.13d)$$

For the case of the deuteron, the above model yields the *exact representation of* $\mu_D{}^{\rm exp}$, Eq. (1.1),

$$\hat{\mu}^{\rm tot}_{\rm theor} = g_p n_{4p}/n_{3p} + g_n n_{4n}/n_{3n} \approx (g_p + g_n) n_4/n_3 \equiv \mu_D{}^{\rm exp} = 0.857 \ , \quad (4.14a)$$

$$n_4 = 1.000 \ , \quad n_3 = 1.000 \times 0.880/0.857 = 1.026 \ , \qquad (4.14b)$$

with consequential ellipsoidical shape of the two nucleons

$$n_3{}^2 = 1.054 \ , \quad n_1{}^2 = n_2{}^2 = (1/n_3{}^2)^{1/2} = 0.974 \ . \qquad (4.15)$$

which is precisely of the *prolate* character, as expected (Sect. 1).

FIGURE 7. **The structure of the deuteron according to relativistic hadronic mechanics.** A schematic view of the structure of the deuteron according to the iso-Dirac equation (4.3) for which the charge distribution of the individual nucleons is deformed into a spheroidal ellipsoid of a *prolate*

type, which implies a *decrease* of the conventional values of the magnetic moments of the individual nucleons when in vacuum. In turn, such a decrease permits the apparently first exact-numerical representation of the total magnetic moment of the deuteron. It should be indicated that the deformation, expressed by Eqs. (4.15), is only of a few percentage points. Yet its conceptual, theoretical and experimental implications are far reaching, as indicated in the final part of this paper.

Note that the different normalization ${n_1}^2 + {n_2}^2 + {n_3}^2 = 3$ would yield the values

$$ {n_3}^2 = 1.054\,, \quad {n_1}^2 = {n_2}^2 = (3 - {n_3}^2)/2 = 0.973 \tag{4.16} $$

which are very close to the preceding ones.

Note that the data for the 4π spinorial symmetry tests, Eq.s (4.10), and those for the deuteron, Eq.s (4.15), are very close. This illustrates that the 4π-interferometric measures, even though not inclusive of strong nuclear forces, could provided experimental evidence on the alterability of the intrinsic magnetic moments of nucleons in the deuteron structure and, therefore, resolve the problem of total nuclear magnetic moments.

Note also that, along the historical hypothesis of Fig. 1, the fit (4.14) is reached via a geometrical representation of the deformation of the charge distribution, which is applicable to any preferred structure model, the opposite approach of adapting the deformation of magnetic moments to any conjectural structural model being manifestly questionable.

We finally note that the representation (4.14) is exact via an isorelativistic treatment in first quantization based on only the D state, while the conventional relativistic treatment [2] uses all possible S-, D- and P-states without achieving such an exact representation.

The following additional applications of HM and its relativistic extension should also be indicated (in addition to those of ref.s [7]):

1) Nuclear physics: Reconstruction of the exact rotational symmetry for deformed-oscillating nuclei [6k]; reconstruction of the exact isospin symmetry in nuclear physics [6h]; axiomatically consistent representation of dissipative nuclear processes [6k]; and others.

2) Particle Physics: reconstruction of the exact Minkowski space,

Poincaré symmetry and special relativity at the isotopic level [6k] for all possible signature-preserving alteration of the flat space-time geometry [16]; exact-numerical representation of the behaviour of the meanlives of unstable hadron with energy [17]; exact iso-Minkowskian representation [17a] and experimental fit [18b] of the Bose-Einstein correlation [18c] for high energy [18d] and low energy [18e] from first axiomatic principles and without *ad hoc and unknown* parameters as originating form the nonlocal-integral interactions due to mutual overlapping of the wavepackets; numerical interpretation of the synthesis of the neutron from protons and electrons only as occurring in stars [6i]; isoquark theory [19] with conventional quantum numbers, exact confinement and convergent perturbative series; reconstruction of the exact parity under weak interactions [6k]; new classical and quantum theory of antiparticles characterized by the antiautomorphic isodual map of conventional classical and quantum theories of matter, $A \rightarrow A^d = -A^\dagger$ [20]; and others.

3) Gravitation and astrophysics: Achievement of the universal invariance of gravitation [6g] which is given by the isopoincaré symmetry of Sect. 3 for the particular case when the iso-Minkowskian metric equals the Riemannian metric, $\hat{\eta}(x, \dot{x}, \ldots) = g(x)$; achievement of an operator form of gravity which is as axiomatically consistent as RQM [21], which is again given by the isopoincaré formulations of Sect. 3 with $\hat{\eta} = g(x)$, since they are of operator character; numerical representation of the large difference in cosmological redshift of quasars when physically attached to their associated galaxies [22a,22b] as due to the decrease of the speed of light within the huge quasars chromosphere represented via the iso-Minkowskian geometry under the isospecial relativity; numerical representation of the internal quasars redshift and blueshift [22c]; new isoselfdual cosmology with equal distribution of matter and antimatter and total null characteristics of the universe [6k]; and others;

4) Superconductivity: achievement of an explicitly *attractive* interaction among the two *identical* electrons of the Cooper pair [23], which is reached via the isotopic lifting of the conventional Coulomb problem outlined in Sect. 3, in excellent agreement with experimental data.

5) Theoretical Biology: Axiomatic representation of irreversible and nonconservative characters of biological systems; identification of the apparent origin of irreversibility at the ultimate level of constituents with nonlocal correlation effects; new geometric representations of locomotion and bifurcations in biological systems; and others [24].

5 Apparent new recycling of nuclear waste

The experimental finalization of the alterability of the intrinsic magnetic moments of nucleons via total nuclear magnetic moments, 4π-interferometric measures, or other means signals a necessary departure from the conventional linear, local-differential and Hamiltonian-unitary formulation of the Poincaré symmetry.

First, the above occurrence would establish the applicability in the nuclear structure of the isopoincaré symmetry, the isospecial relativity and related RHM. Since the latter are *directly universal* for all possible alterations of the geometry of empty space, *they would then apply even when not desired.*

At any rate, the isopoincaré symmetry is the only generalized symmetry known to this author which permits the preservation of the abstract axioms of the special relativity under nonunitary maps, because conventional deformations [4] imply necessary structural departures. As a matter of fact, the abstract identity of the isotopic and conventional symmetries, $\hat{P}(3.1) \equiv P(3.1)$, with consequential preservation under "isotopic completion" of conventional physical laws explains the reasons why their experimental validity, by no means, implies that conventional QM is the only applicable theory.

Once the above elements are understood, implications of the historical hypothesis of Fig. 1 are consequential. The fist consists of apparently new means for recycling nuclear waste which can be used by the nuclear power companies in their own plants, thus avoiding altogether the dangerous and expensive transportation of the waste to yet un-identified dumping site.

In essence, RHM in its nuclear realization and the fundamental isopoincaré symmetry predict the possible mutation not only of the intrinsic magnetic moment of the neutron, but also of its meanlife, to such an extent that the former implies the latter and viceversa (as one can see via the use of the isoboosts). In turn, the control of the meanlife of the neutron *de facto* implies new means for recycling the nuclear waste.

In this respect, the first physical reality which should be noted (and admitted) is that total nuclear magnetic moments constitute *experimental evidence* on the alterability of the inrinsic magnetic moments of nucleons.

The second physical reality which should be noted (and admitted) is that, by no means, the neutron has a constant and universal meanlife, because it possesses a meanlife depending on the local conditions. In fact, the neutron's meanlife is of the order of seconds when belonging to certain nuclei with rapid

beta decays; a meanlife of the order of 15 minutes when in vacuum; a meanlife of the order of days, weeks and years when belonging to other nuclei; all the way to an infinite meanlife for stable nuclei.

Once the above occurrences are admitted, the *basic principle for possible new recycling of nuclear waste is the "stimulated neutron decay" (SND) consisting of resonating or other subnuclear mechanisms suitable to stimulate its beta decay*. Among the various possibilities under study, we quote here the possible *gamma stimulated neutron decay*. (GSND) according to the reaction [25a]

$$\gamma + n \rightarrow p^{+} + e^{-} + \bar{\nu} \; , \tag{5.1}$$

which is predicted by RQM to have a very small (and therefore practically insignificant) cross section as a function of the energy, but which is instead predicted by RHM to have a resonating peak in the cross section at the value of 1.294 MeV (corresponding to 3.129×10^{20} Hz). As such, the above mechanism is of *subnuclear* character, in the sense of occurring in the *structure of the neutron*, rather than in the nuclear structure, the latter merely implying possible refinements of the resonating frequency due to the (relatively smaller) nuclear binding energy [7b].

When stable elements are considered, the above GSND is admitted only in certain instances, evidently when the transition is compatible with all conventional laws. This is the case for the isotope Mo(100,42) which, under the GSND, would transform via beta emission into the Te(100,43) which, in turn, is naturally unstable and beta decays into Ru(100,44). For a number of additional admissible elements see [25a].

The point important for this note is that the GSND is predicted to be admissible for large and unstable nuclei as occurring in the nuclear waste. The possible new form of recycling submitted for study in this note is given by *bombarding the radioactive waste with a beam of photons of the needed excitation frequency and of the maximal possible intensity*. Such a beam would cause an instantaneous excess of peripheral protons in the waste nuclei with their consequential decay due to instantaneous excess of Coulomb repulsive forces.

It should be stressed that this note can only address the basic *principle* of the GSND. Once experimentally established (see later on), the recycling requires evident additional technological studies on the *equipment* suitable

to produce the photon beam in the desired frequency and intensity, e.g., via synchrotron radiation or other mechanisms [25c].

The imporatant point is that equipment of the above nature is expected to be definitely smaller in size, weight and cost than large particle accelerators. As such, the recycling is expected to verify the basic requirement of usability by the nuclear power companies in their own plants.

A novelty of the proposed new recycling is that it is specifically conceived to occur at the *subnuclear* level. A virtually endless number of possibilities exist for the reduction of the meanlife of the waste via mechanisms of *nuclear* type. Among them we note mechanisms based on RQM, such as those by Shaffer et al. [26a], Marriot et al. [26b], Barker [26c] and others, as well as new *nuclear* mechanisms predicted by RHM and currently under patenting. The understanding is that, to maximize the efficiency, the final equipment is expected to be a combination various means of both subnuclear and nuclear character.

6 Possible additional advances

The possible experimental verification of the alterability of the magnetic moment and meanlife of the neutron would rather deep implications throughout all aspects of nuclear physics. In addition to possible new forms of recycling nuclear waste indicated above, it may be of some value to indicate the following additional, possibilities.

1) Nuclear forces. RHM terminates the study of nuclear forces by adding terms and terms in the Hamiltonian, because it provides means for rigorous new studies based on the representation with potentials of terms truly being of action-at-a-distance, and the representation with the isounit of contact, nonlinear, nonlocal and nonunitary effects for which the notion of a potential has no conceptual, mathematical or physical meaning of any nature;

2) Nuclear structure. RHM permits a deeper understanding of the nuclear structure via the admission of small, yet significant interactions of *nonlocal-integral and nonhamiltonian-nonunitary* type, due to the wave-overlapping of the constituents. In turn, it can be safely stated that the inclusion of the latter interactions will inevitably lead to new nuclear mod-

els. New nuclear reactions cannot also be excluded in view of the *attractive* character of the latter interactions under certain rather selective but well identified conditions even against the Coulomb barrier, as theoretically and experimentally established in [23].

3) Controlled fusion. One of the first applications of the studies of this paper is expected for the attempts at reaching a "hot controlled fusion" with a positive energy balance. In fact, these attempts are essentially costituted by a condensation phase caused by magnetic or other means which works as predicted, followed by the initiation of the fusion process with its notorious instabilities which have not been controlled to date.

It is evident that RQM is exactly valid for the condensation phase due to large mutual distances. However, the exact validity of the same discipline at the initiation of the fusion process does not appear to have solid scientific grounds because of the emergence of numerous new effects at short distances, such as those of nonlinear, nonlocal and nonpotential type, which are beyond any hope of quantitative treatment via RQM. In view of evident societal aspects, deeper studies with a covering disciplines appear, therefore, to be warranted.

In particular, the studies of this paper suggest that *the intrinsic magnetic moments of protons and neutrons is expected to change precisely at the initiation of the fusion process*. It is then evident, although not widely admitted, that such a change has implications for the currently uncontrolled instabilities, and should be reflected in a re-design of the magnetic and other confinement. After all, the control of the instabilities is currently conducted under the (tacit) assumption that protons and neutrons preserve their intrinsic magnetic process during the fusion process.

4) New subnuclear energy. Any new recycling of nuclear waste is unavoidably linked to possible new sources of energy. In fact, the GSND

$$\gamma_{\rm res} + {\rm Mo}(100,42) \rightarrow_{\rm stim} {\rm Tc}(100,43) + \beta \rightarrow_{\rm spont} {\rm Ru}(100,44) + \beta \,, \qquad (6.1)$$

is *de facto* a potential new source of *subnuclear* energy called *hadronic energy* [25a] (see also the review [25b]) which releases the rather large amount of about 5 MeV plus the energy would not release harmful radiations, would not imply radioactive waste, would not require heavy shield or critical mass, and would be realizable in large or minuterized forms.

5) Other possible applications. Numerous additional applications are conceivable, such as the production of rare isotopes via GSNT, the prediction of neutron rays for industrial applications via the synthesis of the neutron from protons and electrons beams in flight, medical applications, and others.

7 Proposed experiments

The continuation of quantitative scientific studies on the proposed new recycling of the nuclear waste (as well as on the other applications indicated above) beyond the level of personal views one way or another, requires the following three basic experiments, all of truly fundamental character, moderate cost and full realization with current technology.

I. Finalize the interferometric 4π spinorial symmetry measures [3]. The above measures are manifestly fundamental for possible new forms of recycling as well as for possible new forms of subnuclear energy. In fact, they would provide experimental evidence on possible deviations from the Poincaré symmetry in favor of our covering isopoincaré form. This is due to the fact that, if confirmed, the measures would establish a deviation from the fundamental *spinorial* transformation law in favor of the mutated form (4.8). The alterability of the meanlife of the neutron would then be consequential.

It should be stressed that the primary evidence for the alterability of the intrinsic magnetic moments of nucleons rests in the experimental values of total nuclear magnetic moments. The finalization of interferometric measures [3] would merely provide additional backing on the same alterability. The latter would however occur for controllable conditions, thus being invaluable for other predictions via extrapolations.

II. Repeat don Borghi's experiment [25d] on the apparent synthesis of the neutron from protons and electrons *only*. Despite momentous advances, we still miss fundamental experimental knowledge on the structure of the neutron, e.g., on how the neutron is synthesized from protons and electrons *only* in young stars solely composed of hydrogen (where quark models cannot be used owing to the lack of the remaining members of the baryonic octet, and weak interactions do not provide sufficient information on the *structure* problem).

The synthesis occurs according to the reaction

$$p^+ + e^- \rightarrow n + \nu \, , \tag{7.1}$$

which: is the "inverse" of the stimulated decay (6.1); is predicted by RQM to have a very small cross section as a function of the energy; while the same cross section is predicted by RHM to have a peak at the threshold energy of 0.80 MeV in singlet p-e coupling [6i].

The possible synthesis of the neutron has a fundamental relevance for waste recycling, besides other industrial applications. If the electron "disappears" at the creation of the neutron, as in current theoretical views, the GSND becomes of difficult if not impossible realization.

However, the electron is a permanent and stable particle. As such, doubts as to whether it can "disappears" date back to Rutherford's very conception of the neutron as a "compressed hydrogen atom". As well known, RQM does not permit such a representation of the neutron structure on numerous counts. Nevertheless, the covering RHM has indeed achieved an exact-numerical representation of *all* characteristics of the neutron according to Rutherford's original conception [6i].

The novelty permitting the above result is that, when immersed within the hyperdense proton, the electron experiences a mutation $e^- \rightarrow \hat{e}^-$ of its *intrinsic* characteristics (becoming a quark ?) including its rest energy (because $n_4 \neq 1$ inside the proton, thus $E_{\hat{e}} = mc^2 = m_e {c_0}^2/{n_4}^2$). The excitation energy of 1.294 MeV is predicted by our covering isopoincaré symmetry under the condition of recovering all characteristics of the neutron for the model $n = ({p^+}_\uparrow, {\hat{e}^-}_\downarrow)_{\mathrm{RHM}}$, including its primary decay for which [6i] $\hat{e}^- \rightarrow e^- + \bar{\nu}$.

A preliminary experimental verification of the synthesize the neutron in laboratory was done by don Borghi's and his associates [25d]. Since experiments can be confirmed or dismissed *solely* via other experiments and certainly not via theoretical beliefs one way or the other, don Borghi's experiment must be run again and either proved or disproved. The test can be repeated either as originally done [loc. cit.], or in a number of alternative ways, e.g., by hitting with a cathodic ray of 0.80 MeV a mass of beryllium saturated with hydrogen, put at low temperature and subjected to an intense electric field to maximize the p-e singlet coupling. The detection of neutrons emanating from such a set-up would establish their synthesis.

III. Complete Tsagas' experiment [25e] on the stimulated neutron decay. A most fundamental information needed for additional quan-

titative studies is the verification or disproof of the GSND at the resonating gamma frequency of 1.294 MeV.

The latter experiment has been recently initiated by N. Tsagas and his associates [25e]. It consists of disk of the radioisotope Eu^{152} (which naturally emits gammas of 1.3 MeV) placed parallel and close to a disk of an element admitting of the GSND, such as the Mo(100, 42) (or a sample of nuclear waste). The detection of electrons with at least 2 MeV emanating from the system would establish the *principle* of the GSND (because such electrons can only be of subnuclear origin, Compton electrons being of at most 1 MeV). The detection via mass spectrography of traces of the extremely rare Ru(100,44) after sufficient running time would confirm said principle beyond any reasonable doubt. The practical realization of the proposed form of waste recycling would then be shifted to the industrial development and production of a photon beam of the needed frequency and intensity via a relatively small equipment usable by the nuclear power companies in their own plants.

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